Manhole Storage Capacity Influence on Transient Flow Modeling during Storm Sewer Flooding Event

Mathurin Daynou, Musandji Fuamba and Tew-Fik Mahdi

Drainage systems are designed to quickly convey rainwater from urban areas to areas of natural flow (rivers, streams, lakes, etc). Hence, they are sized with the assumption of steady free surface flow. With the frequent flooding of storm sewers and corresponding damage to public and private infrastructure (Schmitt et al. 2004), numerical modeling of transient flow has now become unavoidable. Several models of transient flow are available, each with its advantages and limitations. This chapter examines the impact of the inclusion of manhole size on the filling celerity of urban drainage systems. It is assumed that the pressurization of the drainage network starts with the rapid filling of a manhole, i.e. as soon as the water level in the manhole reaches the crown of the pipe being filled.

Based on this assumption, a filling model is built to analyze the influence of manhole capacity on the propagation of the surge wave. This model uses the method of characteristics within the conduit and an innovative method based on gravity waves.

To evaluate the impact of manhole size on the filling speed of the Storm Water System (SWS), the proposed methodology is applied to a theoretical case study. Results show that manhole size can accelerate or reduce the filling speed of connecting conduits.

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16.1 Introduction

During extreme flood events, the inflow may exceed the hydraulic capacity of the receiving SWS conduits. Pressurization of the SWS may start when the water level in the manhole reaches the crown of the outlet pipe.

Transient flow modeling in a sewer conduit is closely related to its boundary conditions. By collecting water from upstream pipes and surface runoff, manholes play an important role during the storm sewer filling process. For example, if inflow is higher than outflow, the manhole water level starts to rise. This increase will be more or less rapid depending on the manhole size.

Let's assume that transient flow results from a rapid increase of the water level into the manhole structure. A pressurization/depressurization wave may form from one end of the pipe (up/downstream) and propagate to the other. Hydraulic conditions may also result into formation of two waves at each boundary. These waves can then propagate into opposite directions and interact with each other, thus trapping air into the conduit. Such a phenomenon may occur when the conduit is not properly ventilated.

When simulating different flow types in SWS, manholes are often modeled as junction points (Yen 1986). Depending on the manhole dimension and storage capacity involved, flood routing can be ignored in large manholes thus resulting in an inaccurate simulation of the flow propagation. Manhole dimension should therefore be considered in the numerical modeling when dealing with transient flow simulations in Storm Water Systems.

The purpose of this study is to systematically evaluate manhole influence on mixed flow modeling in SWS, as well as to evaluate the influence of manhole storage capacity on the surge front celerity during an extreme event. In this chapter, we will limit our investigation to a surge front propagating in the downstream direction under the sole influence of the upstream manhole.

16.2 Methodology

The studied system includes an upstream manhole and a downstream manhole connected by a pipe. Let's assume that the filling surge starts from the upstream manhole moving downstream (Figure 16.1). The complete study includes the flow conditions inside the pipe, the upstream boundary condition (flow condition at the connection between the upstream manhole

and the pipe), and the downstream boundary conditions (flow condition at the connection between the pipe and the downstream manhole).

Figure 16.1 Studied system.

16.2.1 Flow Conditions Inside the Pipe

According to Yen (1986), Wiggert (1972), Song et al. (1983), Li and McCorquodale (1999), Hamam and McCorquodale (1982), and Fuamba (1997; 2002) among others, the first regime a sewer pipe may experience during an extreme flood event is the free-surface flow regime characterized by gravity flow and a water depth smaller than the pipe diameter. If the incoming flow is higher than the outgoing flow, the water depth increases quickly and may reach the sewer crown at the upstream junction. A surge front will then develop and propagate inside the pipe thus resulting in the simultaneous existence of a free surface and a pressurized flow regime. Once the surge front reaches the downstream boundary, the sewer pipe becomes pressurized. Following Fuamba (1997; 2002), the modeling technique proposed in this chapter is dynamic and each flow regime is handled by a specific application.

Free Surface Flow Conditions

Free surface flow will be modeled by a set of nonlinear partial differential equations known as the St.Venant equations. They reflect the principles of continuity and momentum conservation in the flow:

$$\begin{cases} D_h \frac{\partial U}{\partial x} + U \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = 0 \\ \frac{1}{g} \frac{\partial U}{\partial t} + \frac{U}{g} \frac{\partial U}{\partial x} + \frac{\partial h}{\partial x} = J_0 - J_e \end{cases} \quad (16.1)$$

Where U is the mean cross-sectional velocity, h the water depth, D_h the hydraulic depth, g the acceleration due to gravity, x the distance along the pipe, t the time, J_0 the sewer bottom slope and n the Manning coefficient.

$J_e = \frac{n^2 U |U|}{R_h^{\frac{4}{3}}}$ represents the energy grade line and R_h is the hydraulic radius.

Using the characteristics technique, the system of partial differential equations (Equation 16.1) can be transformed into the following two systems of ordinary differential equations, each one valid along the characteristic line it describes:

$$C^+ \begin{cases} \frac{dU}{dt} + \frac{g}{c} \frac{dh}{dt} = g(J_f - J_e) \\ \frac{dx}{dt} = U + c \end{cases} \quad C^- \begin{cases} \frac{dU}{dt} - \frac{g}{c} \frac{dh}{dt} = g(J_f - J_e) \\ \frac{dx}{dt} = U - c \end{cases} \quad (16.2)$$

These equations are integrated through the method of specific intervals (Graf and Altinakar 2000; Lai 1986; Lister 1961).

Figure 16.2 Method of specified intervals.

It is assumed (Figure 16.2) that at time t , flow variables h and U are known at successive nodes $(i-1)$, (i) and $(i+1)$ of a regular grid. An attempt is made to calculate flow variables for the next time step $(t + \Delta t)$ at a point (P) located on the same vertical (i) . Assuming that the values of c and J_e calculated at L and R respectively remain valid along the characteristic lines, we may transform Equation (16.2) as follows:

$$C^+ \begin{cases} (U_p - U_L) + \frac{g}{c_L}(h_p - h_L) = g[J_0 - (J_e)_L] \Delta t \\ x_p - x_L = (U_L + c_L) \Delta t \end{cases} \quad (16.3)$$

$$C^- \begin{cases} (U_p - U_R) - \frac{g}{c_R}(h_p - h_R) = g[J_0 - (J_e)_R] \Delta t \\ x_p - x_R = (U_R - c_R) \Delta t \end{cases} \quad (16.4)$$

From positive characteristic (C^+) and negative characteristic (C^-) we get the following equations:

$$U_p + \frac{g}{c_L} h_p = U_L + \frac{g}{c_L} h_L + g[J_f - (J_e)_L] \Delta t = K_{pos} \quad (16.5)$$

$$U_p - \frac{g}{c_R} h_p = U_R - \frac{g}{c_R} h_R + g[J_f - (J_e)_R] \Delta t = K_{neg} \quad (16.6)$$

where: $\Delta t = t_p - t_L = t_p - t_R$, $(J_e)_{R/L} = n^2 U_{R/L} |U_{R/L}| [(R_H)_{R/L}]^{-\frac{4}{3}}$, and

$$(R_H)_{R/L} = \frac{D}{4} \left[1 - \frac{\sin\left(2 \cos^{-1}\left(1 - \frac{h_{R/L}}{D}\right)\right)}{2 \cos^{-1}\left(1 - \frac{h_{R/L}}{D}\right)} \right]$$

Solving for U_p and h_p results in:

$$U_p = \frac{c_L K_{pos} + c_R K_{neg}}{c_R + c_L} \quad (16.7)$$

$$h_p = \frac{c_R c_L}{g(c_L + c_R)} (K_{pos} - K_{neg}) \quad (16.8)$$

In Equations (16.7) and (16.8), it is assumed that point L lies between nodes (i) and (i+1). To calculate the velocity, the water depth and the celerity at L, it is assumed that flow parameters vary linearly between nodes (i) and (i+1). Therefore, one can interpolate

$$\frac{U_i - U_L}{U_i - U_{i-1}} = \frac{c_i - c_L}{c_i - c_{i-1}} = \frac{h_i - h_L}{h_i - h_{i-1}} = \frac{\Delta t}{\Delta x} (U_L + c_L) \quad (16.9)$$

and deduce the flow variables (velocity, celerity and water depth) at L :

$$U_L = \frac{\Delta x U_i - \Delta t (c_{i-1} U_i - c_i U_{i-1})}{\Delta x + \Delta t (U_i - U_{i-1} + c_i - c_{i-1})} \quad (16.10)$$

$$c_L = \frac{\Delta x c_i - \Delta t U_L (c_i - c_{i-1})}{\Delta x + \Delta t (c_i - c_{i-1})} \quad (16.11)$$

$$h_L = h_i - \frac{\Delta t}{\Delta x} (U_L + c_L) (h_i - h_{i-1}) \quad (16.12)$$

Calculation of flow variables at point R depends on the flow regime. R will be situated among nodes (i) and (i+1) for a subcritical flow, at node (i) for a critical flow and between (i-1) and (i) for a supercritical flow. The interpolation is made in the same way as described above. Results are presented in Table 16.1 below:

Table 16.1 Calculations of flow variable at R .

SUBCRITICAL FLOW		$U_R = \frac{\Delta x U_i - \Delta t (c_{i+1} U_i - c_i U_{i+1})}{\Delta x - \Delta t (U_i - U_{i+1} - c_i + c_{i+1})}$ $c_R = \frac{\Delta x c_i + \Delta t U_R (c_i - c_{i+1})}{\Delta x + \Delta x (c_i - c_{i+1})}$ $h_L = h_i - \frac{\Delta t}{\Delta x} (U_R - c_R) (h_i - h_{i+1})$
SUPERCritical FLOW		$U_R = \frac{\Delta x U_i + \Delta t (U_i c_{i-1} - c_i U_{i-1})}{\Delta x + \Delta t (U_i - U_{i-1} - c_i + c_{i-1})}$ $c_R = \frac{\Delta t U_R (c_i - c_{i-1}) - \Delta x c_i}{\Delta t (c_i - c_{i-1}) - \Delta x}$ $h_R = h_i - \frac{\Delta t}{\Delta x} (U_R - c_R) (h_i - h_{i-1})$

Mixed Flow Conditions

Following previous studies (Fuamba 2002; Guo and Song 1990; Song et al. 1983; Wiggert 1972), the Shock-Fitting technique is used to compute flow variables (velocity, depth and pressure) for the mixed flow regime. While the St. Venant

equations are used to represent the flow conditions in the free surface portion of the pipe, an equivalent formula provided by Wylie and Streeter (Wylie and Streeter 1980) is used to model the flow conditions in the pressurized pipe zone:

$$\begin{array}{cc} \text{Free Surface Flow} & \text{Pressurized Flow} \\ \left\{ \begin{array}{l} \frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x} + \frac{c^2}{g} \frac{\partial U}{\partial x} = 0 \\ \frac{1}{g} \frac{\partial U}{\partial t} + \frac{U}{g} \frac{\partial U}{\partial x} + \frac{\partial h}{\partial x} = J_f - J_e \end{array} \right. & \left\{ \begin{array}{l} \frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x} + \frac{a^2}{g} \frac{\partial U}{\partial x} = 0 \\ \frac{1}{g} \frac{\partial U}{\partial t} + \frac{U}{g} \frac{\partial U}{\partial x} + \frac{\partial h}{\partial x} = J_f - J_e \end{array} \right. \end{array} \quad (16.13)$$

The “Shock-Fitting” technique is used to locate the pipe surge front at each time step, by solving the system of nonlinear equations based on the conservation of mass and momentum across the front.

Figure 16.3 Tracking procedure.

Considering Figure 16.3, B is the surge front location at time t and P is its position at time $t + \Delta t$. Furthermore, conditions of pressurized and free surface flow are known at positions A and C, respectively. Six unknown variables are to be determined in a section located in the transitional zone: the front position (x_I), the velocity (V_2) and water depth (y_2) in the free surface portion of the flow, the velocity (V_I) and water depth (h_I) in the pressurized portion of the flow and the moving front velocity (w). A system of six equations is necessary to determine these unknowns:

Two equations are based on mass and momentum conservation across the front:

$$g(A_2 \bar{y}_2 - A_1 h_1) = A_1 (U_1 - w)(U_1 - U_2) \quad (16.14)$$

$$A_1(U_1 - w) = A_2(U_2 - w) \quad (16.15)$$

The third equation describes the surge front propagation: denoting by l the distance traveled by the surge front between positions B and P , this equation can be expressed by:

$$dl = w dx \quad (16.16)$$

The characteristic lines C_2^+ and C_2^- which are valid throughout the free surface flow domain are used as the fourth and fifth equations:

$$(U_p - U_R)C_R + g(h_p - h_R) + gC_R(S_R - S_0)\Delta t = 0 \quad (16.17)$$

$$(U_p - U_S)C_R + g(h_p - h_S) + gC_R(S_S - S_0)\Delta t = 0 \quad (16.18)$$

The sixth and last equation is given by the positive characteristic applicable on the pressurized side of the surge front:

$$h_p = h_{R_i} + U_{R_i} \left(\frac{a}{g} - \frac{f\Delta t}{2gD} |U_{R_i}| \right) - \frac{a}{g} U_p \quad (16.19)$$

The calculation procedure at the surge interface is iterative until the convergence criterion is satisfied. The surge front is initialized assuming an initial celerity of the surge front 10 times greater than the last calculated free surface wave celerity (Fuamba 1997; 2002). This value will be reduced or increased inside the iterative procedure to match the real surge front celerity.

Fully Pressurized Flow Conditions

For fully pressurized flow, the following positive and negative characteristic equations (Fuamba 1997) are used to compute the flow variables:

$$h_p = h_R + U_R \left(\frac{a}{g} - \frac{f\Delta t}{2gD} |U_R| \right) - \frac{a}{g} U_p \quad (16.20)$$

$$h_p = h_S - U_S \left(\frac{a}{g} - \frac{f\Delta t}{2gD} |U_S| \right) + \frac{a}{g} U_p \quad (16.21)$$

where h is the pressure head, f the Darcy coefficient and a the water hammer celerity express by Finnemore and Franzini (2002) as:

$$a = \sqrt{\left[\rho \left(\frac{1}{E_v} + \frac{1}{tE} \right) \right]^{-1}} \quad (16.22)$$

In which ρ is the density of water, E_v is the bulk modulus of elasticity of the medium, E is the modulus of elasticity of the material and t the conduit thickness.

16.2.2 Upstream Boundary Conditions

Upstream boundary conditions are determined at the interface between the pipe and the manhole. To come as close as possible to real flow conditions, the proposed method should be able to take into account both flow regimes (subcritical and supercritical) and specific physical conditions that may prevail inside pipes during the filling process of the sewer. From this point of view, four specific cases (Figure 16.4) are discussed in detail when dealing with the boundary flow simulation.

The “emptying phase” (decreasing of the manhole water level) is treated separately from the “filling phase” (increasing of the water level in the manhole). Each case is associated with the two main flow regimes (subcritical and supercritical). The overall strategy of the boundary conditions calculation is to solve variables (head at junction, flow and head at the conduit upstream end) using available equations (continuity, energy conservation and characteristic equation).

Figure 16.4 Classification of different flow configurations at the upstream boundary.

Upstream Boundary Conditions for Subcritical Flows

It is assumed that the manhole total inflow is related to upstream pipe inflow (Q_{amt}^t) and catchment basin inflow (Q_{reg}^t) as:

$$Q_E^t = Q_{amt}^t + Q_{reg}^t \quad (16.23)$$

The total inflow at time $t + \Delta t$ is denoted by $Q_E^{t+\Delta t}$ and Q_E^t is the total inflow at time t . The mean inflow volume in the upstream manhole between time t and $t + \Delta t$ can be expressed as:

$$\Delta V_E = (Q_E^{t+\Delta t} + Q_E^t) \frac{\Delta t}{2} \quad (16.24)$$

and the mean outflow volume as:

$$\Delta V_S = (Q_S^{t+\Delta t} + Q_S^t) \frac{\Delta t}{2} \quad (16.25)$$

where Q_S^t and $Q_S^{t+\Delta t}$ are the outflows at time t and $t + \Delta t$ respectively. The volume balance between time t and $t + \Delta t$ can then be evaluated by:

$$\Delta V = \Delta V_E - \Delta V_S = (Q_E^{t+\Delta t} + Q_E^t - Q_S^{t+\Delta t} - Q_S^t) \frac{\Delta t}{2} \quad (16.26)$$

The manhole water level will increase by ΔH_{REG} if the balance is positive or will decrease by ΔH_{REG} in the contrary. This variation of the water level is evaluated hereafter:

$$\Delta H_{REG} = \frac{\Delta V}{S_{REG}} = \frac{\Delta t}{2S_{REG}} (Q_E^{t+\Delta t} + Q_E^t - Q_S^{t+\Delta t} - Q_S^t) \quad (16.27)$$

Furthermore, the future manhole water level $H_{REG}^{t+\Delta t}$ is related to the present manhole water level H_{REG}^t as follows:

$$H_{REG}^{t+\Delta t} = H_{REG}^t + \Delta H_{REG} \quad (16.28)$$

Assuming that the manhole acts as an upstream reservoir with negligible flow velocity, we can write (Chaudhry 1993):

$$H_{COND}^{t+\Delta t} = H_{REG}^{t+\Delta t} - \frac{(1+k)}{2g} (U_{COND}^{t+\Delta t})^2 \quad (16.29)$$

Substituting $H_{REG}^{t+\Delta t}$ into Equation (16.27), we obtain:

$$\begin{aligned} H_{COND}^{t+\Delta t} &= H_{REG}^t + \Delta H_{REG} - \frac{(1+k)}{2g} \left(U_{COND}^{t+\Delta t} \right)^2 \\ &= H_{REG}^t + \frac{\Delta t}{2S_{REG}} \left(Q_E^{t+\Delta t} - Q_E^t - Q_S^{t+\Delta t} + Q_S^t \right) - \frac{(1+k)}{2g} \left(U_{COND}^{t+\Delta t} \right)^2 \end{aligned} \quad (16.30)$$

where k is the loss coefficient at the junction between manhole and pipe, g is the gravity acceleration and $U_{COND}^{t+\Delta t}$ represents the flow velocity at the pipe entrance for the next time step.

For a circular pipe, flow rate is related to the wetted perimeter and velocity:

$$Q_S^{t+\Delta t} = U_{COND}^{t+\Delta t} S_M = U_{COND}^{t+\Delta t} \frac{R^2}{2} (\theta - \sin \theta) \quad (16.31)$$

where R is the pipe radius and θ is given by:

$$\theta = 2 \arccos \left(1 - \frac{H_{COND}^{t+\Delta t}}{R} \right) \quad (16.32)$$

Now, Equation (16.31) becomes:

$$Q_S^{t+\Delta t} = U_{COND}^{t+\Delta t} \frac{R^2}{2} \left(2 \arccos \left(1 - \frac{H_{COND}^{t+\Delta t}}{R} \right) - \sin \left(2 \arccos \left(1 - \frac{H_{COND}^{t+\Delta t}}{R} \right) \right) \right) \quad (16.33)$$

Substituting Equation (16.33) into Equation (16.30), we obtain a single equation with two unknowns: $H_{COND}^{t+\Delta t}$ and $U_{COND}^{t+\Delta t}$. The second equation needed to solve the system is obtained by considering the negative characteristic:

$$U_{COND}^{t+\Delta t} - \left(\frac{g}{C_R} \right) H_{COND}^{t+\Delta t} = K_{neg} \quad (16.34)$$

The solution of the final system (16.35) to be solved for unknowns H and U is obtained through a combination of the Newton-Raphson method and the bisection method (Press 1992).

$$\begin{cases} H = H'_{REG} + \frac{\Delta t}{2S_{REG}} \left(\frac{Q_E^{t+\Delta t} - Q_E^t + Q_S^t - \dots}{2} \left(2 \cos \left(1 - \frac{H}{R} \right) - \dots \right) \right) - (1+K) \frac{U^2}{2g} \\ U - \left(\frac{g}{C_R} \right) H = K_{neg} \end{cases} \quad (16.35)$$

Upstream Boundary Conditions for Supercritical Flows

For a supercritical flow, the method presented above cannot be used since positive and negative characteristics are excluded from the solution domain. An innovative method based on gravity waves (Chaudhry 1993; Graf and Altinakar 2000) is proposed below. This method is based on the following simplifying assumptions: (i) the component of the weight of the liquid in the downstream direction is equal to the shear force acting on the channel sides and bottom; (ii) the pressure distribution on both sides of the wave front is hydrostatic; (iii) the velocity distribution is uniform on both sides of the wave front; (iv) the wave is an abrupt discontinuity of negligible length; (v) the wave shape does not change as it propagates along the channel; (vi) the water surface behind the wave is parallel to the initial water surface.

Assuming that the manhole can be considered as a storage junction, the conservation of mass can be written as:

$$I - O = \frac{dS}{dt} \quad (16.36)$$

where I and O are respectively the manhole inflow and outflow discharges, S is the junction storage, and dS is the change in storage during the time interval dt . Assuming that I and O are time varying functions, one can approximate dS/dt by $\Delta S/\Delta t$. Equation (16.36) can now be rewritten as:

$$I\Delta t - O\Delta t = \Delta S \quad (16.37)$$

If the subscripts 1 and 2 are used to indicate values at times t and $t + \Delta t$ respectively, then Equation (16.37) can be written as:

$$\frac{1}{2}(I_1 + I_2)\Delta t - \frac{1}{2}(O_1 + O_2)\Delta t = S_2 - S_1 \quad (16.38)$$

While I_1 , I_2 , O_1 and S_1 are known at time t , values for the next time step O_2 and S_2 are unknown. Assuming a uniform junction cross section A , S_1 and S_2 can be related to the water depth in the manhole as:

$$S_1 = H_1 A ; S_2 = H_2 A \quad (16.39)$$

and Equation (16.38) becomes:

$$(I_1 + I_2) \Delta t - (O_1 + O_2) \Delta t = 2(H_2 A - H_1 A) \quad (16.40)$$

Moreover, the outflow is related to the pipe velocity V_2 and wetted section A_2 as:

$$O_2 = V_2 A_2 \quad (16.41)$$

and the wetted section can be written as a function of the water depth:

$$A_2 = \frac{R^2}{2} (\theta_2 - \sin \theta_2) = \frac{R^2}{2} \left\{ 2 \cos \left(1 - \frac{h_2}{R} \right) - \sin \left[2 \cos \left(1 - \frac{h_2}{R} \right) \right] \right\} \quad (16.42)$$

where R is the radius of the circular sewer conduit given by Equation (16.32).

Using the energy conservation principle (Yen 1986), we can relate the manhole water depth H to the pipe water depth h as:

$$H_2 = h_2 + (1-k) \frac{V_2^2}{2g} ; H_1 = h_1 + (1-k) \frac{V_1^2}{2g} \quad (16.43)$$

where k is the junction loss coefficient, V the mean cross-sectional velocity and g the gravitational acceleration.

Substituting Equations (16.43) and (16.42) into Equation (16.40) and rearranging, we obtain the following function of two unknowns:

$$f(h_2, U_2) = h_2 - h_1 + \frac{1-k}{2g} (V_2^2 - V_1^2) + \dots$$

$$\left\{ \frac{V_2 R^2}{2} \left[2 \cos \left(1 - \frac{h_2}{R} \right) - \sin \left(2 \cos \left(1 - \frac{h_2}{R} \right) \right) \right] - I_1 - I_2 + O_1 \right\} \frac{\Delta t}{2A} \quad (16.44)$$

To find a solution to the problem, we need a second equation. For this purpose, we use the gravity wave theory. The inflow increase at the pipe entrance can be assumed sudden and results in a moving wave with absolute celerity V_w as shown in Figure 16.5 below. By applying velocity V_w on the

entire system in the upstream direction, we are able to convert unsteady-flow situation into a steady-flow one.

Figure 16.5 Filling phase of the manhole.

Applying mass conservation and momentum principle between sections A_1 and A_2 , we obtain the following equations:

$$(V_w - V_1)A_1 = (V_w - V_2)A_2 \quad (\text{continuity}) \quad (16.45)$$

$$\frac{A_1}{g}(V_1 - V_w)(V_1 - V_2) = A_2\bar{y}_2 - A_1\bar{y}_1 \quad (\text{momentum}) \quad (16.46)$$

in which A_1 and A_2 are the wetted perimeters, V_1 and V_2 are the flow velocities, g is the gravitational acceleration, $\bar{y}_1$ and $\bar{y}_2$ are the centroids of wetted surfaces A_1 and A_2 that can be calculated as follows:

$$\bar{y}_n = \begin{cases} \frac{2}{3A_n} \left[R \cdot \sin \left(\arccos \left(1 - \frac{h_n}{R} \right) \right) \right]^3 - R + h_n & \text{if } (h_n \leq R) \\ \frac{2}{3A_n} \left[R \cdot \sin \left(\arccos \left(\frac{h_n}{R} - 1 \right) \right) \right]^3 - R + h_n & \text{if } (h_n \geq R) \end{cases} \quad (16.47)$$

The velocity of the surge wave front can be obtained from Equations (16.45) and (16.46) as:

$$V_w = V_1 + \sqrt{A_2 g \frac{(A_2\bar{y}_2 - A_1\bar{y}_1)}{A_1(A_2 - A_1)}} \quad (16.48)$$

Substituting Equation (16.48) into Equation (16.45) and rearranging, we obtain the following function of two unknowns (V_2 and h_2):

$$g(V_2, h_2) = A_1 A_2 (V_1 - V_2)^2 - g(A_2\bar{y}_2 - A_1\bar{y}_1)(A_2 - A_1) \quad (16.49)$$

To obtain flow conditions for the next time step, we need to solve the following system of non linear equations:

$$\begin{cases} f(V_2, h_2) = 0 \\ g(V_2, h_2) = 0 \end{cases} \quad (16.50)$$

where $f(V_2, h_2)$ is given by Equation (16.44) and $g(V_2, h_2)$ by Equation (16.49). The solution is obtained by using the globally convergent method for nonlinear systems of equations (Press et al. 1995).

During the emptying phase, the total inflow is less than the total outflow, thus resulting in a lowering of the manhole water level. Using the same technique as presented above we obtain the configuration presented in Figure 16.6.

Figure 16.6 Emptying phase of the manhole.

In this case, applying the same method results into the final velocity function:

$$g(V_2, h_2) = A_1 A_2 (V_1 - V_2)^2 - g(A_1 \bar{y}_1 - A_2 \bar{y}_2)(A_1 - A_2) \quad (16.51)$$

Finally, Equation (16.50) will be solved to compute flow variables.

16.2.3 Downstream Boundary Conditions

For simplification purposes, the downstream manhole water level is always assumed lower than the upstream pipe water level so as to avoid flow reversal. Therefore, the downstream flow condition will be calculated for subcritical and supercritical conditions as illustrated in Figure 16.7.

Figure 16.7 Downstream boundary flow configurations.

Downstream Boundary Conditions for Subcritical Flows

For a subcritical flow, positive characteristic Equation (16.5) will be used together with the subcritical flow velocity equation to compute flow variables (velocity and water depth) at the downstream boundary:

$$\begin{cases} U_p + \left(\frac{g}{c_L}\right) h_p = K_{pos} \\ U_p = \frac{1}{n} \left(\frac{D}{4} \left\{ 1 - \sin \left[2 \cos^{-1} \left(1 - \frac{h_p}{D} \right) \right] \right\} \left[2 \cos^{-1} \left(1 - \frac{h_p}{D} \right) \right]^{-1} \right)^{\frac{2}{3}} J_e^{\frac{1}{2}} \end{cases} \quad (16.52)$$

This system is solved through a combination of the Newton-Raphson and bisection methods (Press 1992).

Downstream Boundary Conditions for Supercritical Flows

For supercritical flow, both negative and positive characteristics are in the solution domain and can be used to solve the flow equations. From the positive characteristic we obtain the following equations representing the flow variables at the n and $n-1$ downstream sections (see Figure 16.7):

$$U_L = \frac{\Delta x U_n - \Delta t (c_{n-1} U_n - c_n U_{n-1})}{\Delta x + \Delta t (U_n - U_{n-1} + c_n - c_{n-1})} \quad (16.53)$$

$$c_L = \frac{\Delta x c_n - \Delta t U_L (c_n - c_{n-1})}{\Delta x + \Delta t (c_n - c_{n-1})} \quad (16.54)$$

$$h_L = h_n - \frac{\Delta t}{\Delta x} (U_L + c_L) (h_n - h_{n-1}) \quad (16.55)$$

and the following ones from the negative characteristic:

$$U_R = \frac{\Delta x U_n + \Delta t (U_n c_{n-1} - c_n U_{n-1})}{\Delta x + \Delta t (U_n - U_{n-1} - c_n + c_{n-1})} \quad (16.56)$$

$$c_R = \frac{\Delta t U_R (c_n - c_{n-1}) - \Delta x c_n}{\Delta t (c_n - c_{n-1}) - \Delta x} \quad (16.57)$$

$$h_R = h_n - \frac{\Delta t}{\Delta x} (U_R - c_R) (h_n - h_{n-1}) \quad (16.58)$$

The boundary condition (water depth and velocity) can then be easily evaluated with:

$$U_n = \frac{c_L K_{pos} + c_R K_{neg}}{c_R + c_L} \quad (16.59)$$

$$h_n = \frac{c_R c_L}{g(c_L + c_R)} (K_{pos} - K_{neg}) \quad (16.60)$$

where K_{pos} and K_{neg} are given by:

$$K_{pos} = U_L + \frac{g}{c_L} h_L + g [J_f - (J_e)_L] \Delta t \quad (16.61)$$

$$K_{neg} = U_R - \frac{g}{c_R} h_R + g [J_f - (J_e)_R] \Delta t \quad (16.62)$$

16.2.4 Calculation Procedure

It is assumed that the boundary flow conditions (velocity and water depth) are known at every grid point of the solution domain at time t . The calculation procedure is time-marching. At each time step, the upstream boundary is first calculated. Then, the flow condition inside the pipe and the downstream boundary condition are computed. The complete procedure is as follows:

1. The initial flow condition is computed assuming an initial free surface flow inside the SWS: sewer water depth and manhole water depth are computed accordingly, assuming a steady uniform flow.
2. The upstream manhole water level is calculated through a balance in volume between incoming and outgoing flows for time step Δt in the upstream manhole.
3. The new manhole water level is then compared to the previous one to determine if the situation is a “filling” or an “emptying” of the manhole. Appropriate equations are used for each case. Calculation of the pipe entrance flow conditions are then achieved accordingly.
4. Free-surface flow equations are used to calculate the flow conditions in the pipe for the next time step.

5. The downstream boundary condition is calculated at the last section of the pipe.
6. Steps one through four are repeated for every time step until the water level in the pipe is at least equal to the diameter of the pipe, which concludes the free surface phase.
7. For the mixed phase, the surge front is initialized assuming an initial celerity of the surge front 10 times greater than the last calculated wave celerity (Fuamba 1997; 2002).
8. The water level in the manhole at any time step is determined by establishing the balance in volume between inflow and outflow during the considered time step. A tracking technique is used to follow the front wave as it travels downstream.
9. When the wave front reaches the downstream end, the pipe is now fully pressurized and the mixed phase ends giving way to the pressurized phase.

16.3 Application and Discussions

16.3.1 Case Study

The application of the methodology outlined above is carried out on a hypothetical case study which aims to verify the ability of the proposed model to correctly simulate the flow conditions during the rising limb and the falling limb of the inflow hydrograph. The hypothetical system is formed by a 100-m long circular pipe with a 1-m diameter, connecting two manholes. To analyze the manhole storage capacity influence on free surface flow simulation in SWS, the discharge hydrograph used at the upstream manhole is assumed to be triangular and the peak discharge less or equal to the maximum uniform flow in the pipe. The upstream manhole size is then increased virtually at every program run and the filling time (time taken by the manhole water level to reach the pipe crown) is computed.

Thereafter, the influence of the manhole storage capacity on the time taken by the pressurization wave to propagate from the upstream manhole to the downstream boundary is investigated. In this case, the flow rate entering into the upstream manhole is determined by a triangular hydrograph whose peak is twice the maximum uniform flow in the pipe. The time taken by the

surge front to propagate on the entire pipe length is computed for a 1-m manhole diameter and for a 2-m manhole diameter.

16.3.2 Preliminary Results and Discussion

Figure 16.8 below compares the incoming and outgoing hydrographs as well as the water level variation into the manhole and at the conduit entrance. It can be seen that for a 1-m diameter, the incoming hydrograph is similar to the outgoing one. The water level variation inside the manhole and at the conduit entrance are also comparable, except at high flow rates. However, when the diameter increases (from one to four m), one can clearly see the flow routing effect and its consequences on the water level inside the manhole and at the pipe entrance.

Figure 16.8 Influence of the manhole size on the free surface boundary flow simulation.

Let's consider the time taken by the upstream manhole water level to reach the crown of the outlet pipe. For that purpose, the peak discharge of the

inflow hydrograph is increased so that the water level in the manhole can reach the crown of the sewer pipe. As illustrated in the Figure 16.9 below, the filling time (time for the manhole water level to reach the outlet conduit crown) is related to manhole diameter. Starting with a 1-m manhole, the filling time is about 104 seconds after which an extra time lag is added for each 1-m increase in manhole diameter: The filling times are about 160 seconds for a 2-m diameter, 167 seconds for a 3-m diameter, and 125 seconds for a 4-m diameter.

Figure 16.9 Manhole storage capacity influence on the filling time during free surface boundary flow simulation.

Finally, the influence of the manhole storage capacity on the conduit pressurization is assessed along with the wave front celerity. The time taken by the surge wave to reach the downstream boundary section is simulated for a manhole diameter of one and two m. Figure 16.10 (a and b) illustrates the propagation of the surge front. For a 1-m diameter manhole, the time taken by the wave front to reach the downstream boundary is about 4 sec,

whereas it is about 5.4 sec when the diameter is 2-m. In addition, the upstream manhole water level is approximately one m higher in the 1-m diameter manhole than the water level in a 2-m manhole. This difference is significant especially during flooding events.

Figure 16.10 Propagation of the surge front from the upstream manhole to the downstream boundary.

16.4 Conclusion

In this chapter, a method to handle complex boundary flow conditions in storm sewers is proposed. This method shows that it is possible to integrate manhole size in a dynamic simulation of transient flow in storm sewers. The proposed method uses an innovative method based on the gravity wave theory and several solving techniques, each one specific to a particular flow condition. The present study is limited to one surge front moving downstream and does not consider the influence of the downstream manhole which is still under investigation.

When applied to a theoretical case study, the method is found to be successful since flow conditions are correctly simulated during the rising limb and the falling limb of the inflow hydrograph. The influence of the manhole size on flow simulation in a Storm Water System is clearly illustrated for free surface, mixed and pressurized flow.

Furthermore, this approach is an opportunity to combine several solving techniques in a dynamic calculation of the SWS that could increase the efficiency of the simulation model.

The present work is part of a global research which aims to integrate upstream and downstream manhole effects into the transient air-water flow

simulation of SWS. The results presented in this chapter are partial and research continues.

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References

- Chaudhry, M. H. (1993). *Open-channel flow*, Prentice Hall, Englewood Cliffs, NJ.
- Finnemore, E. J., and Franzini, J. B. (2002). *Fluid mechanics with engineering applications*, 10th Ed., McGraw-Hill, Boston ; Montreal.
- Fuamba, M. (1997). "Transient Flows Modelling in Drainage Systems Networks (in French)," Ph.D. dissertation, Katholieke Universiteit Leuven, Belgium.
- Fuamba, M. (2002). "Contribution on transient flow modelling in storm sewers." *Journal of Hydraulic Research*, 40(6), 685-693.
- Graf, W. H., and Altinakar, M. S. (2000). *Hydraulique fluviale : écoulement et phénomènes de transport dans les canaux à géométrie simple*, 2e éd. corrigée Ed., Presses polytechniques et universitaires romandes, Lausanne, [Suisse].
- Guo, Q., and Song, C. C. S. (1990). "Surging in urban storm drainage systems." *Journal of Hydraulic Engineering*, 116(12), 1523-1539.
- Hamam, M. A., and McCorquodale, J. A. (1982). "Transient conditions in the transition from gravity to surcharged sewer flow." *Canadian Journal of Civil Engineering*, 9(2), 189-196.
- Lai, C. (1986). "Numerical Modelling of unsteady open-channel flow." In: *Advances in hydroscience*, A. Press, ed., New York, NY, 161-333.
- Li, J., and McCorquodale, A. (1999). "Modeling mixed flow in storm sewers." *Journal of Hydraulic Engineering*, 125(11), 1170-1180.
- Lister, M. (1961). "Method of Characteristics." In: *Numerical Methods for Computers*, W. Fox, ed., New York, NY.
- Press, W. H. (1992). *Numerical recipes in C : the art of scientific computing*, 2nd Ed., Cambridge University Press, Cambridge [Angleterre].
- Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P. (1995). *Numerical recipes in C : the art of scientific computing*, 2nd Ed., Cambridge University Press, Cambridge [Angleterre].
- Schmitt, T. G., Thomas, M., and Ettrich, N. (2004). "Analysis and modeling of flooding in urban drainage systems." *Journal of Hydrology*, 299(3-4), 300-311.
- Song, C. C. S., Cardle, J. A., and Kim Sau, L. (1983). "Transient mixed-flow models for storm sewers." *Journal of Hydraulic Engineering*, 109(11), 1487-1504.

- Wiggert, D. C. (1972). "Transient flow in free- surface, pressurized systems." ASCE J Hydraul Div, 98(HY1), 11-27.
- Wylie, E. B., and Streeter, V. L. (1980). "Multidimensional Fluid Transients by Latticework." Journal of Fluids Engineering, Transactions of the ASME, 102(2), 203-210.
- Yen, B. C. (1986). "Hydraulics of Sewers." Advances in Hydrosience, 14, 1-122.