Modeling the Reliability of Water Distribution Systems

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This chapter demonstrates that the performance of a municipal water distribution system at a given time can be measured in terms of reliability. Such a measure will be very useful in developing new maintenance and rehabilitation strategies. To find the reliability measures the distribution system is converted to a stochastic network. The reliability values at various nodes of the network are computed using either the method of minimal pathsets or the method of minimal cutsets. The relative merits of the two methods are discussed and guidelines on the choice of the appropriate method are given. A complete theoretical development of the underlying methodology is presented and the procedure for converting a given water distribution system into a stochastic network is outlined. A case study network is analyzed to illustrate the theoretical developments.

12.1 Introduction

At present our water infrastructure, especially in the older cities, is in critical stages of deterioration and has started to crumble. Due to an out-of-sight, out-of-mind philosophy and the lack of funds to follow a preventive maintenance practice, the replacement is mostly performed on a react-to-crisis basis. In spite of this supposedly conventional wisdom, this may not be the best strategy since substantial expenditure and inconvenience can be

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avoided by replacing a deteriorated pipeline before it actually breaks. The conventional reactive practice could also be attributed to a lack of quantitative documentation to convince the decision-makers of the urgency of a preventive maintenance program (Shamsi, 2002).

The diagnosis of the degree of deterioration and predicting future breaks remained a mystery until the early eighties. Recent research (Walski, 1993; Walski, 1993a; Quimpo and Shamsi, 1991; Shamsi, 1990; Mays, 1989; Male et al., 1990; Shamsi, 1988; Quimpo and Shamsi, 1987) has unfolded new ways and means of modeling the effect of pipe breaks on the reliability of water supply. Briefly, just as a hydraulic model can simulate future flows and pressures in a water distribution system, a probabilistic model can compute future deterioration, pipe breaks, and reliability of water supply. Since pipes make up the largest portion of a water distribution system, this chapter will focus on pipes only. However, the proposed methodology is also applicable to other distribution system components such as, valves, pumps, and tanks. For example, Walski (1993) shows how to include valves in the reliability analysis of water distribution systems.

Most municipal water supply systems are composed of subsystems such as a source (or sources) of water, bulk water pumping and transmission facilities, treatment plant, finished water storage and a distribution network of pipelines. Each subsystem consists of components. For example distribution network consists of pipelines, joints, valves, pumps and tanks etc. As the system ages, the components start breaking and the water supply is interrupted at certain demand points. Since the failure of the components is random, the water supply at a demand point is also random and can be expressed only in terms of an associated probability.

Reliability theory, a technique that has evolved rapidly since its adoption by the military during World War II and its extremely successful application to the space and electronics industry, is equally applicable to quantify the performance of a water distribution system in time. The prediction of the future performance of the distribution system is very important from the preventive maintenance point of view. In an environment where competition for municipal resources is fierce, water authority managers need quantitative documentation to convince the decision makers of the urgency of a preventive maintenance program for a system which for the most part is hidden underground. Thus far, due to lack of a scientific rationale for preventive maintenance, the problem is approached in the following two manners: (i) react-to-crisis approach, in which the components are fixed only after they break; (ii) routine maintenance procedures, based on the rules of thumb, such as leaks are detected and fixed every third year, valves are

examined and repaired annually etc. The first approach results in excessive repair costs and the inconvenience which could have been avoided if the action was taken before the crisis. The second approach arbitrarily results in less or more maintenance effort than is actually required.

System Reliability is defined as the probability that a system performs its mission, under specified conditions, during a specified time period. In the present context, assuming the mission to be the transportation of water from the supply sources to all the demand points, we define water distribution system reliability at a given demand point, as the probability that water from a given source (or sources) of supply is available at a given time, at the given demand point. Quantification of the system's performance in terms of reliability is expected to open the horizons for maintenance and rehabilitation strategies based on sound scientific principles. The basic idea is to compute system reliability periodically and perform preventive maintenance or replacement whenever the reliability falls below a threshold level adopted by the utility.

The reliability at a demand point depends on the reliability of components which comprise different paths from the supply source to the demand point. If all the paths are closed then water will not be available at all. The probability that at least one such path is open from the source to the demand point, is the reliability we want to compute. Path closure may be caused by component failures such as pipe breaks or valve failure. Hence, component reliabilities can be used to determine the reliabilities at the demand points. To compute component reliabilities a regression model (Shamir and Howard, 1979 and Clark et al., 1982) is developed for each component or a group of similar components, which gives the number of breaks in time, as a function of certain variables. In case of pipelines such variables may be age, diameter, length, material, pressure, soil type, temperature, landcover, traffic load etc. Andreou et al. (1987) suggest Cox's regression model for reliability computations at the individual pipe level. This paper assumes that models are available to compute the reliability of system components with time.

A Stochastic Network is a set of points (referred to as nodes) that are interconnected by a set of directed lines (referred to as arcs), each of which either functions or fails according to some probability distribution. This chapter proposes that a water distribution system can be represented as a stochastic network in which arcs represent the subsystems or components and nodes represent the demand points or the junctions between the components. The reliability at nodes, then can be computed using any of the network reliability analysis techniques, developed already in reliability

engineering. For large and complex networks Hwang et al. (1981) recommend method of minimal pathsets and cutsets. Tung (1985) recommends minimal cutset analysis for water distribution networks. Quimpo and Shamsi (1987) suggest that, once the reliability has been computed at all the demand points, a reliability surface may be drawn for the network analogous to the surface of hydraulic heads that is commonly calculated for a distribution system. Low points in reliability surface will identify sectors of the distribution system which need maintenance or rehabilitation.

12.2 Definitions

Examples for the following definitions are given from the network shown in Figure 12.1.

Source: A node that has arcs only leaving the node and no arcs entering the node. Example: node 1.

Sink: A node that has arcs only entering the node and no arcs leaving the node. Example: node 16.

Pathset: A pathset between nodes u and v of a network, denoted by $P^*(u,v)$, is a set of arcs $\{a_1, a_2, a_3, \dots\}$ which forms a connection from u to v . Example: $P^*(1,6) = \{1, 4, 5\}$.

Minimal Pathset: A pathset is minimal if it contains no subset which is also a pathset. It is denoted by $P(u,v)$. Example: $P(1,6) = \{1,5\}$.

Cutset: A cutset between nodes u and v of a network, denoted by $C^*(u,v)$, is a set of arcs whose removal (or failure) disconnects u from v . Example: $C^*(1,6) = \{1, 4, 8\}$.

Minimal Cutset: A cutset is minimal if it contains no subset which is also a cutset. It is denoted by $C(u,v)$. Example: $C(1,6) = \{1,4\}$.

Order: The order of a minimal pathset or cutset is the number of arcs contained in it.

Standard Network: A network is in standard form if it has the following properties:

1. it has only one source;
2. it has only one sink;
3. the arc failures are independent, i.e., the life times of the components are statistically independent;
4. all nodes are perfectly reliable;

5. there are no parallel arcs between any two nodes; and
6. there are no loops, i.e., a path originating at one node does not terminate or pass through the same node.

The first four assumptions are made to simplify theoretical developments for reliability computations, while the last two are generally required by most minimal pathset and cutset enumeration algorithms. The network shown in Figure 12.1 is in standard form.

Figure 12-1. Hypothetical water distribution network.

2.3 Theory

Two approaches are available for modeling the distribution system reliability. The first approach, pioneered by Dr. R.G. Quimpo of the University of Pittsburgh (Quimpo and Shamsi, 1987) considers the probability of pipe breaks and source-to-node connectivity to compute the probability of water availability regardless of its flow or pressure. This approach examines whether or not any water will be available. The second approach (Wagner et al., 1988) determines the probability of a specific flow or pressure using analytical or simulation methods. Although the second approach is more practical and sophisticated, its application has been limited to simple series-parallel reducible networks due to computational complexities. While series-parallel reductions are a significant and valuable tool in evaluation of network reliability, there are few series-parallel reducible water systems (Goulter and Jacobs, 1989). In large cities where pipe breaks have become a daily nuisance, an uninterrupted water supply is much more critical than the availability of ideal flows and pressures and can be analyzed by using the first approach.

For a stochastic network, node-pair reliability or NPR, at time t , denoted by $R_{u,v}(t)$ is defined as the probability that node u can communicate with another node v in the network, at time t . We shall call the pair (u,v) as the node-pair. Initially, we assume that network is in standard form. The case of a general network will be considered in the next section where the procedure for converting a given network into standard form is described. In a water distribution network, NPR is exemplified by the probability that a demand point represented by node v receives water from the source node represented by node u . If node-pair is not specified, u and v are supposed to be source and sink nodes, respectively, and NPR is called network reliability.

The NPR analysis results presented in this chapter were conducted using a FORTRAN computer program called RAPACK (Reliability Analysis Package). The theoretical background and description of the RAPACK software can be found in the author's dissertation (Shamsi, 1988).

12.3.1 Minimal Pathset and Cutset Enumeration

The first step in the NPR evaluation technique described in this chapter, is the enumeration of all the minimal pathsets and cutsets between the given node-pair. Numerous enumeration techniques have been developed in the fields of reliability, networks and graph theory. Some algorithms enumerate

minimal pathsets first and then derive minimal cutsets from them. Other algorithms enumerate both of them simultaneously. The best algorithm is one which is easy to program, needs simple input information about the structure of the network, has least restrictions on the type and shape of the network and requires least storage and CPU time. This work used an algorithm developed by Inoue and Henley (1975) and applied by Pearson (1977) in his availability computation program. The algorithm is of simultaneous enumeration type. The input information consists of specifying number, initial node and terminal node for all the arcs of the network.

12.3.2 Exact NPR

This section develops the equations for computing the exact value of NPR at a given time t .

Minimal Pathset Analysis

Let $P_1, P_2, \dots, P_m$ be m minimal pathsets between the given node-pair, at time t , and define the events $E_1, E_2, \dots, E_m$ by

$$E_i = \{\text{All arcs in } P_i \text{ function}\} \quad (12.1)$$

Since arc failures are assumed to be independent, a minimal pathset functions if all the arcs contained in it function, hence

$$Pr\{E_i\} = \prod_{r \in P_i} R_r(t) \quad (12.2)$$

$R_r(t)$ being the reliability (function probability) of the r -th arc at time t . The node-pair communicates if and only if all the arcs of at least one minimal pathset function or at least one of the events E_i occur. Hence the NPR at time t can be expressed by

$$\begin{aligned} R_{u,v}(t) &= Pr \{ \text{at least one minimal pathset functions at time } t \} \\ &= Pr \{ E_1 \text{ or } E_2 \text{ or } \dots E_m \} \end{aligned}$$

$$R_{u,v}(t) = Pr \left\{ \bigcup_{i=1}^m E_i \right\} \quad (12.3)$$

where $\cup$ denotes the union. By use of the expansion rule for the probability of the union of m events, we can expand Equation 12.3 as

$$R_{u,v}(t) = \sum_{i=1}^m Pr\{E_i\} - \sum_{i<j} Pr\{E_i \cdot E_j\} + \sum_{i<j<k} Pr\{E_i \cdot E_j \cdot E_k\} + \dots + (-1)^{m-1} Pr\{E_1 \dots E_m\} \quad (12.4)$$

where $Pr\{\bullet\}$ denotes the probability and

$$Pr\{E_i \cdot E_j\} = \prod_{r \in P_i \cup P_j} R_r(t) \quad (12.5)$$

and so on. The NPR expression is then given in terms of arc reliabilities by

$$R_{u,v}(t) = \sum_{i=1}^m \prod_{r \in P_i} R_r(t) - \sum_{i<j} \prod_{r \in P_i \cup P_j} R_r(t) + \sum_{i<j<k} \prod_{r \in P_i \cup P_j \cup P_k} R_r(t) + \dots + (-1)^{m-1} \prod_{all r} R_r(t) \quad (12.6)$$

Minimal Pathset Analysis

Let $C_1, C_2, \dots, C_n$ be n minimal cutsets between the node-pair, at time t , and define the events $F_1, F_2, \dots, F_n$ by

$$F_i = \{All \text{ arcs in } C_i \text{ fail}\} \quad (12.7)$$

A minimal cutset fails if all the arcs contained in it fail, hence

$$Pr\{F_i\} = \prod_{r \in C_i} 1 - R_r(t) = \prod_{r \in C_i} Q_r(t) \quad (12.8)$$

$Q_r(t)$ being the unreliability (failure probability) of the r -th arc at time t . The node-pair is disconnected if and only if all the arcs of at least one of the minimal cutsets are failed or if at least one of the events F_i occur. Hence the node-pair unreliability (NPU) at time t can be expressed by

$$\begin{aligned} UR_{u,v}(t) &= Pr\{at \text{ least one minimal cutset fails at time } t\} \\ &= Pr\{F_1 \text{ or } F_2 \text{ or } \dots F_n\} \end{aligned}$$

$$UR_{u,v}(t) = 1 - R_{u,v}(t) = Pr\left\{\bigcup_{i=1}^n F_i\right\} \quad (12.9)$$

Expanding Equation 12.9 as before, we get

$$R_{u,v}(t) = 1 - \sum_{i=1}^n Pr\{F_i\} + \sum_{i<j} Pr\{F_i \cdot F_j\} - \sum_{i<j<k} Pr\{F_i \cdot F_j \cdot F_k\} + \dots + (-1)^{n-1} Pr\{F_1 \dots F_n\} \quad (12.10)$$

where:

$$Pr\{F_i \cdot F_j\} = \prod_{r \in C_i \cup C_j} 1 - R_r(t) = \prod_{r \in C_i \cup C_j} Q_r(t) \quad (12.11)$$

and so on. The NPR expression is then given in terms of arc unreliabilities by

$$R_{u,v}(t) = 1 - \sum_{i=1}^n \prod_{r \in C_i} Q_r(t) + \sum_{i<j} \prod_{r \in C_i \cup C_j} Q_r(t) - \sum_{i<j<k} \prod_{r \in C_i \cup C_j \cup C_k} Q_r(t) + \dots + (-1)^{n-1} \prod_{all\ r} Q_r(t) \quad (12.12)$$

12.3.3 Approximate NPR

Equations 12.6 and 12.12 give exact value of NPR. In each equation there are m and n summation terms respectively, of the form $\sum IIR_r(t)$ or $\sum IQ_r(t)$ separated by alternating + or - signs. Each summation term consists of the individual event terms, corresponding to the combination of events E_i or F_i .

In the x-th summation term, there are $\binom{m}{x}$ event terms if Equation 12.6 is

used or $\binom{n}{x}$ event terms if Equation 12.12 is used. The total number of

event terms in each equation are $(2m - 1)$ and $(2n - 1)$ respectively. For complex systems, m or n may be large and it will be uneconomical to use the exact expression for $R_{u,v}(t)$. Successive upper and lower bounds can be obtained by using fewer summation terms. For example in case of minimal analysis, including one, two and three summation terms, the bounds can be expressed as

$$U_1 = \sum_{i=1}^m \prod_{r \in P_i} R_r(t) \quad (12.13)$$

$$L_1 = \sum_{i=1}^m \prod_{r \in P_i} R_r(t) - \sum_{i < j} \prod_{r \in P_i \cup P_j} R_r(t) \quad (12.14)$$

$$U_2 = \sum_{i=1}^m \prod_{r \in P_i} R_r(t) - \sum_{i < j} \prod_{r \in P_i \cup P_j} R_r(t) + \sum_{i < j < k} \prod_{r \in P_i \cup P_j \cup P_k} R_r(t) \quad (12.15)$$

and so on. Similarly, the minimal cutset form of the bounds, using one and two summation terms can be expressed as

$$L_1 = 1 - \sum_{i=1}^n \prod_{r \in C_i} Q_r(t) \quad (12.16)$$

$$U_1 = 1 - \sum_{i=1}^n \prod_{r \in C_i} Q_r(t) - \sum_{i < j} \prod_{r \in C_i \cup C_j} Q_r(t) \quad (12.17)$$

and so on. Each summation term, when added algebraically to the previous summation terms, gives a new bound to the NPR, and therefore we shall call these terms as bounding terms. Therefore, if exact value of reliability is not required and an approximate value within a user specified interval called accuracy parameter (say ± 0.01) is acceptable, all the bounding terms might not have to be evaluated. In such a case, the computation of bounding terms can be stopped if the difference between the current and the previous bounding term is less than the accuracy parameter.

12.3.4 Choice between Minimal Pathset and Minimal Cutset Analysis

From above discussion it is obvious that, though either Equation 12.6 or Equation 12.12 can be used to compute exact NPR, the number of event terms and hence the CPU time would be different in each case. We must choose the method which takes less CPU time.

A knowledge about the redundancy of the network can help in selecting one of the two methods. If the network is highly redundant, the number of minimal pathsets will be more than the number of minimal cutsets, and

hence minimal cutset analysis will be computationally more efficient. On the other hand, if the network is not as highly redundant, the chances of number of minimal cutsets being greater than the number of minimal pathsets are more prominent, and in such a case, minimal pathset analysis will be suitable. Water distribution networks are usually not highly redundant, since laying down too many redundant pipelines is very expensive. Hence, minimal pathset analysis will be computationally more efficient for exact NPR computations in general. The redundancy of the network can be estimated by enumerating minimal pathsets and cutsets at some selected demand points of the network. However, the points selected should be well within the system (i.e. far from the supply sources), because redundancy is minimum in the neighborhood of the supply sources.

However, if bounds rather than an exact value of NPR is being computed, the choice can not be based merely on m or n , since we never know in advance that how many bounding terms would have to be computed before the accuracy criterion is satisfied. It is quite possible that computation of only a few bounding terms would satisfy the accuracy criterion, thus saving CPU time. On the other hand, it is also possible that even the computation of all the bounding terms would not satisfy the accuracy criterion, and hence exact NPR would be computed anyway, without any savings in CPU time.

Hence, the factors affecting the choice between the minimal pathset or minimal cutset analysis, are rather complex, when bounds are being evaluated, since they depend on network structure and the reliability characteristics of its arcs. The reliability engineers therefore established certain rules of thumb, which were based on their experience with numerous systems and networks. Messinger and Shooman (1967) recommend that the bounds based on the minimal cutsets are best in the high reliability region, and those based on the minimal pathsets are best in the low reliability region. However, when a high-reliability system has long series structure (thus having many minimal cutsets and only a few minimal pathsets), the bounds using minimal pathset analyses are sometimes best. This happens to be the case in water distribution networks as well. Pearson (1977), after analyzing numerous systems whose every component has reliability greater than 0.8, suggest that if $n < m^2$, then minimal cutset analysis is suitable, otherwise, minimal pathset analysis is suitable.

12.3.5 Preparing a Stochastic Network From a Distribution System

The following stepwise procedure describes how a given water distribution system can be converted to a stochastic network in the standard form, so that the theoretical developments described in this chapter would be applicable.

1. On the layout plan of the distribution system, identify all the components in the distribution system. Pipelines of different materials and different diameters must be treated as separate components, since they are expected to follow different failure patterns.
2. Mark the direction of flow through all the components. This can be conveniently done, if the distribution system has a computerized hydraulic model. A typical output of such hydraulic models is the magnitude and direction of flow through each component. The directions of flow would identify the sources and sinks in the system from topological point of view. For example, a reservoir is usually a supply source. However from topological point of view it may sometimes act as a sink. It will be a source if water goes out of it, and a sink if water only goes into it.
3. Fix some demand points throughout the system, where reliability has to be computed. These points may be industries, fire hydrants or simply a function of two mains.
4. On a separate sheet of paper, draw a network, representing all components by arcs and the junction points of components by nodes. Demand points must be represented by nodes.
5. In between the demand points, identify the series and parallel assemblies of the arcs and replace them by single equivalent arcs. Suppose there are k arcs in series with each other, with reliabilities $R_1, R_2, \dots, R_k$. The reliability of the equivalent arc R_e is given by

$$R_e = \prod_{i=1}^k R_i \quad (12.18)$$

If k arcs are in parallel with each other, R_e is given by

$$R_e = 1 - \prod_{i=1}^k (1 - R_i) \quad (12.19)$$

6. **Node Failures:** If all the nodes in the network are assumed to be perfectly reliable, the paths between two given nodes will consist of arcs only. The assumption of perfectly reliable nodes may be valid at the source nodes if the water is available all the time. It may be valid at some demand points like fire hydrants, since they are not underground and do not break very often due to routine maintenance procedures. It is valid at the nodes which do not represent physical entities such as dead ends of the pipelines. However, the nodes which do represent physical entities, such as joints and fittings etc., are subject to breaking, and hence can not be treated as perfectly reliable nodes. In such a case, the paths between nodes are a combination of arcs and nodes, and the network is not in the standard form. This problem can be handled in two ways. First one is a graphical approach in which each unreliable node is replaced by an arc with the same reliability. This is shown in Figure 12.2, where an unreliable node j with reliability R is replaced by an arc j_1j_2 with reliability R_j . Second approach is algorithmic in nature, in which the reliabilities of all the arcs originating from node j are multiplied by R_j , before reliability computations.
7. **Multiple Sources and Sinks:** The occurrence of multiple sources is quite possible, since the system may be taking water from a reservoir, a river, and an aquifer. Similarly, multiple sinks such as dead ends of the pipes are totally unavoidable in real networks. Their existence again violates the standard form of the network. This problem can be handled using the following procedure. Suppose that the network of k nodes contains multiple sources $1, 2, \dots, i$ and multiple sinks, $j, j+1, \dots, k$. Then the network can be augmented with a new hypothetical source node 0 and a new hypothetical sink node $k+1$ with hypothetical arcs $(0,1), (0,2), \dots, (0,i)$ and $(j,k+1), (j+1,k+1), \dots, (k,k+1)$ all functioning with reliability one (Shogan, 1974).

8. The network is now in standard form. The arcs and nodes should be numbered next. Most minimal pathset and cutset enumeration algorithms require Terminal Numbering Convention or TNC. In this convention, the numbers are assigned to nodes and arcs in such a way that the numbering begins at the source and continues in such a way that the terminal node of each arc(node) is assigned a number greater than the number used for its initial end. Care should be taken that no two nodes (arcs) receive the same number.

Figure 12.2 Handling unreliable nodes.

12.4 Algorithm

The stepwise algorithm based on the preceding theoretical developments can be summarized as given below.

1. Transform the given water distribution system into a stochastic network in standard form.
2. Define the node-pair (u,v) such that u is the source node (i.e. $u=1$) and v is a demand point where NPR must be computed.

3. Enumerate minimal pathsets (m in number) and minimal cutsets (n in number) for the node-pair (u,v).
4. If exact value of NPR is required, choose minimal pathset analysis and go to step six. Otherwise, go to next step.
5. If $n < m$ choose minimal cutset analysis, else choose minimal pathset analysis.
6. Set $I=1$.
7. Evaluate I-th bounding term.
8. Evaluate (I+1)-th bounding term.
9. If accuracy criterion is satisfied, or if $I=m$ (for minimal pathset analysis), or if $I=n$ (for minimal cutset analysis) then go to next step; else set $I=I+1$ and go to step seven.
10. If $I=m$ or $I=n$, computed value of NPR is exact, else it is approximate.

12.5 Case Studies

12.5.1 Example No. 1

Consider the hypothetical water distribution network shown in Figure 12.1. This network has 20 minimal pathsets and 330 minimal cutsets between the source and the sink. If minimal pathset analysis is performed, there will be

20 bounding terms. The numbers of event terms are $\binom{20}{1} = 20$, $\binom{20}{2} = 190$,

$\binom{20}{3} = 1140$, ... , and so on, in the bounding term number 1, 2, 3, ... , and

so on. The computation of network reliability by successive evaluation of all the twenty bounding terms (about a million event terms since $2^{20} - 1 = 1048575$) is shown in Table 12.1. The convergence to the exact network reliability value of 0.9523 is graphically illustrated in Figure 12.3. On the other hand, if minimal cutset analysis was performed, 330 bounding terms consisting of about $2^{330} - 1 = 2.2 \times 10^{99}$ event terms would have to be evaluated, to compute exact value, which would be wasteful of computer resources.

Table 12.1 Exact network reliability computations using minimal cutset analysis.

Bounding Term Number	Number of Event Terms	Value of Bounding Term	Reliability	
			Value	Status
1	20	10.1669	10.1669	Upper bound
2	190	-61.4008	-51.2340	Lower bound
3	1140	265.8966	214.6627	Upper bound
4	4845	-891.3822	-676.7195	Lower bound
5	15504	2393.0670	1716.3474	Upper bound
6	38760	-5239.4402	-3523.0927	Lower bound
7	77520	9459.6486	5936.5558	Upper bound
8	125970	-14183.7985	-8247.2427	Lower bound
9	167960	17734.1102	9486.8675	Upper bound
10	184756	-18515.7908	-9028.9233	Lower bound
11	167960	16123.7280	7094.8047	Upper bound
12	125970	-11663.7700	-4568.9653	Lower bound
13	77520	6958.7586	2389.7933	Upper bound
14	38760	-3385.9642	-996.1708	Lower bound
15	15504	1312.5479	325.3771	Upper bound
16	4845	-403.6966	-78.3195	Lower bound
17	1140	92.9535	14.6339	Upper bound
18	190	-15.1686	-0.5347	Lower bound
19	20	1.5635	1.0288	Upper bound
20	1	-0.0765	0.9523	Exact
$\Sigma =$		1048575		

Now suppose that an approximate value rather than an exact value of network reliability was required, within an accuracy parameter of $k = 0.01$. It is obvious from Table 12.1 that the accuracy parameter is never satisfied during the computations and all the bounding terms are evaluated. It also shows that approximation is not always possible. However, if minimal cutset analysis is performed, three bounding terms consisting of about 6 million event terms are evaluated, to give a lower bound equal to 0.9519. The summary of computations is given in Table 12.2. It is interesting to note that, the approximate value of NPR was obtained, using the method of

minimal cutsets in about 10 minutes of CPU time (VAX/VMS main frame), as against the exact value of reliability, using the method of minimal pathsets in about 5 minutes of CPU time. The reason is obviously, the evaluation of a larger number of event terms (six million as against one million) in the former case. This example also makes the point that the approximate evaluation does not necessarily guarantee savings in CPU time, as has often been stated in the reliability literature.

Figure 12-3. Convergence to exact NPR by successive evaluation of bounding terms.

Table 12.2 Approximate network reliability computations using minimal analysis.

Bounding Term No.	No. of Event Terms	Value of Bounding Term	Reliability Value	Reliability Status
1	330	-0.0514	0.9486	lower bound
2	54,285	0.0045	0.9531	upper bound
3	5,935,160	-0.0012	0.9519	lower bound *
Total	5,989,775			

* Computations stopped since convergence criteria are satisfied.

12.5.2 Partial Cutset Analysis

Another way of minimizing CPU time, is to do cutset analysis, without enumerating all the minimal cutsets, i.e. if the network contains 300 minimal cutsets, then include only 250 selected cutsets in the analysis, such that the actual value of reliability is only slightly altered. Nelson et al. (1970), for example include the cutsets of up to order five in their computer program and Tung (1985) recommends the first order cutsets. This method is called partial cutset analysis.

Partial Cutset Analysis vs. CPU Time

As shown in Table 12.3, the case study network has a total of 330 minimal cutsets between the source and the sink, out of which there are only eight minimal cutsets of order eight. Including all the 330 cutsets, the reliability has been computed to be 0.95187, which took about 10 minutes of CPU time. However, instead of enumerating all the minimal cutsets, if only the cutsets of up to order six are enumerated, about the same value of reliability is obtained, only in about 5.5 minutes of CPU time. Furthermore, including the minimal cutsets of order 5 only the reliability value is obtained with an error of $0.95199 - 0.95187 = 0.00012$, and consuming only about 49 seconds of CPU time. The tremendous savings of about 9 minutes of CPU time at the loss of a fraction of information, overwhelmingly support the idea of partial cutset analysis. (CPU time reported here, does not include the CPU time spent in enumerating minimal pathsets and cutsets. It only includes the time required for evaluating the event terms and their summation according to Equation 12.12.)

Table 12.3 Partial cutset analysis vs. CPU time.

Order of Cutsets	No. of Cutsets	Cumulative Number of Cutsets	Reliability	CPU Time Minutes: Seconds
2	2	2	0.96159	00:0.02
3	4	6	0.95567	00:0.07
4	42	48	0.95255	00:1.76
5	96	144	0.95199	00:48.58
6	138	282	0.95188	05:23.27
7	40	322	0.95187	08:8.01
8	8	330	0.95187	09:55.35

Figure 12-4. Parallel system.

Partial Cutset Analysis vs. Arc Reliabilities

The question arises: What should be the order of the cutsets, so that CPU time is minimum without a significant error in reliability computations? The answer to such a question will depend on the arc reliabilities. If the arc reliabilities are higher, even a lower order will be able to give satisfactory results. However, if arc reliabilities are comparatively lower, a higher order might be required. We shall illustrate this point with the simple parallel system shown in Figure 12.4, having 7 arcs, each with same reliability of 0.95. This network obviously has only one minimal cutset consisting of all the seven arcs, since the system will fail only if all the seven arcs fail to function. Second column of Table 12.4 shows the network reliability values, as a function of order of the cutset. Fixing an allowable error of 0.0001, it is obvious that the cutset of order three is sufficient to give satisfactory results. However, if the arc reliabilities were 0.90 and 0.85 (columns three. and four in Table 12.4 respectively), the order of the cutset required to attain the same level of accuracy will jump to 4 and 5 respectively.

Partial Cutset Analysis vs. Number of Cutsets

In the parallel system analyzed above, there was only one cut set. We now consider some parallel-in-series systems. Figure 12.5, shows, for example, a 2-parallel-in-series system, consisting of 2 parallel systems, each with seven arcs, connected in series with each other. We thus have 2 minimal cutsets of order seven. Similarly, we shall consider 10 n-parallel-in-series systems

($n=1, \dots, 10$) with n minimal cutsets each. The objective is to see the combined effect of number of minimal cutsets and the order of the cutsets on reliability computations. Concurrently, we shall see the effect of arc reliabilities also. The results of the analysis are presented graphically in Figures 12.6 to 12.8. Each figure shows 10 curves and each curve represents a different n -parallel-in-series network. The numbers from 1 to 10, written on the left ends of all the curves, represent the value of n .

Table 12.4 Partial cutset analysis vs. arc reliabilities.

Order of Cutsets	Network Reliability		
	$R_i=0.95$	$R_i=0.90$	$R_i=0.85$
7	1.0000000	0.9999999	0.9999983
6	1.0000000	0.9999990	0.9999886
5	0.9999997	0.9999900	0.9999241
4	0.9999938	0.9999000	0.9994938
3	0.9998750	0.9990000	0.9966250
2	0.9975000	0.9900000	0.9775000
1	0.9500000	0.9000000	0.8500000

Comparing the three figures, we notice that as the arc reliabilities decrease, we need a higher order of cutsets, to accurately estimate the network reliability. This fact was already established above for a single parallel system. It is also obvious that, larger is the number of minimal cutsets in a network, the higher should be the order of the cutsets included in the computations, since the computational error introduced in the event terms accumulates as the number of minimal cutsets and hence the number of bounding terms increases. For example, Figure 12.8 depicts that, if there was just one minimal cutset, the cutsets of up to order three, will be sufficient to produce the accurate results. However, if there were 10 such minimal cutsets, the cutsets of up to order five will have to be included in the analysis to attain the same level of accuracy.

The above discussion reveals the fact that partial cutset analysis may appreciably reduce the CPU time. However, arbitrary choice of the order of cutsets (such as Nelson et al. and Tung)^{11,5} is not reasonable. The only way to determine the appropriate order of the cutsets is by trial, as shown in Table 12.3, which will of course cost CPU time. Also since the factors

which control CPU time, are network specific, partial cutset analysis does not seem suitable for large systems like water distribution networks.

Figure 12-5. Two parallel-in-series systems

Figure 12-6. Partial cutset analysis vs. number of cutsets ($R_i=0.95$).

Figure 12-7. Partial cutset analysis vs. number of cutsets ($R_i=0.90$)

Figure 12-8. Partial cutset analysis vs. number of cutsets ($R_i=0.85$)

Figure 12-9. Case study water distribution system.

Figure 12-10. Reliability surface before system improvements.

Figure 12-11. Reliability surface after system improvements.

12.5.3 Example No. 2

Another case study water distribution system is shown in Figure 12.9. This system is taken from an example problem from the User's Manual of KYPINES, the legacy water distribution modeling program (Wood, 1980). This system represents the high-pressure region for a municipal water supply system. There is one reservoir, one storage tank, one pump, two check valves, 17 pipes, and 14 junction nodes. Pipes range in size from 6 to 24 inches. Water is pumped from the reservoir located in the western part of the system through a 24-inch transmission main. The storage tank, located on the eastern side of the system provides the fire protection storage for an industrial complex.

The RAPACK software was used to analyze the network and list the pathsets and cutsets between the nodes and the source(s). For example, there are following four pathsets between the source and Node No. 14:

$$P1 = \{1 \ 2 \ 3 \ 15 \ 16\}$$
$$P2 = \{1 \ 5 \ 9 \ 11 \ 17\}$$
$$P3 = \{1 \ 2 \ 12 \ 17\}$$
$$P4 = \{1 \ 5 \ 9 \ 13 \ 14 \ 16\}$$

The first pathset P1 shows that one way for Node No. 14 to receive water from the reservoir (node 1) is through a path formed by pipes 1, 2, 3, 15, and 16, and so on.

Next, the RAPACK software was used to compute NPR values for all the nodes of the network. The point reliability values at the nodes were used to create a contour map of NPR values called a Reliability Surface. Figure 12.10 shows the reliability surface of the case study system for the year 2010. The reliability surface shows the effect of distribution system deterioration on water supply. Low reliability areas represent locations most sensitive to system deterioration and should be given due priority in preventive maintenance and rehabilitation projects. Figure 12.10 shows that reliability is maximum near the source, an intuitively sound result typical of every reliability surface. The eastern half of the distribution system has less than 20% reliability indicating that in the year 2010 water will be available only 20% of the time in the eastern half of the system.

Suppose that the water utility wishes to implement a capital improvement program for water infrastructure rehabilitation. An auxiliary objective is to improve water supply for a future hospital site in the vicinity of Node No. 14. The availability of limited funds requires that maximum water system improvement be achieved at the lowest possible cost. The question, therefore, is which pipe should be replaced to maximize the system improvement and water supply at Node No. 14? Pipe No. 1 is the longest and the largest diameter pipe in the system and its replacement will be the most expensive. Replacement of Pipe No. 1 will also require additional piping work at the pump station and installation of a new check valve. As a second alternative, consider Pipe No. 2 which is known to have been subject to high water pressures of 118 psi and is prone to pipe breakage caused by high pressure induced stresses. Suppose management decides to replace Pipe No. 2. This action will increase the year 2010 pipe reliability by about 59% (0.41 to 0.65). The improvement in Node No. 14 supply reliability is about 35% (0.116 to 0.157). The improved reliability surface is shown in Figure 12.11. It should be noted that by replacing a pipe in the far western part of the system, reliability did not only increase at Node No. 14 but also in the entire eastern half of the distribution system.

12.6 Conclusions and Recommendations

Every water distribution system can be converted into a stochastic network, in which arcs represent the system components and the nodes represent the demand points. Either the method of minimal pathsets or the method of minimal cutsets can be used to compute the reliability at each node of the network. Computation of the exact reliability values costs excessive CPU time and hence an approximate value within a given accuracy may be computed, which usually offers considerable savings in CPU time. Nevertheless, for some networks exact computations may be less expensive. For exact computations, method of minimal pathsets is suitable. For approximate computations the choice between the two methods is rather arbitrary. It is noticed that partial cutset analysis is not suitable.

This work focuses on reliability of water supply availability regardless of its flow or pressure. The proposed methodology might be improved if node reliabilities are defined in terms of probability of flows and pressures. Further research is recommended to enable modeling improved reliability performance measures that include flow and pressure reliabilities.

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