Chapter 10

Sensitivity and Cross-Calibration of Two Types of Continuous Stormwater Models

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Stormwater quality and quantity modeling remains an important activity in current water resources engineering practice. At the same time, the application of stormwater management models is shifting from event to continuous analysis. With this shift, attention is necessarily focused on other forms of model output or performance criteria; hence, the nature of model calibration is quite different in continuous and event modeling applications.

This study involved the comparison of two distinct continuous analysis approaches, continuous simulation and analytical probabilistic modeling. Factorial experimental design was used to test the sensitivity of runoff quantity control measures to various model input parameters. A rudimentary calibration exercise was undertaken in an attempt to correlate the analytical model results to parallel continuous simulation results. The results produced a sufficient calibration procedure, but in the process many questions relating to model assumptions, sensitivity and response were encountered.

10.1 Introduction

Modern computing has offered the ability to analyze large rainfall data sets and undertake calculation-intensive continuous modeling. These analyses are exceptional because they can include the full range of meteorological conditions

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unlike their predecessors the Rational Method and design event modeling approaches. The StormWater Management Model or SWMM (Huber and Dickinson, 1988) was introduced in 1971 as an event simulation model, and later enhanced to continuous modeling. Other event based models, including ILLUDAS (Stall and Terstriep, 1979), HYMO (Williams and Hann, 1973) and OTTHYMO (Wisner et al., 1989), have been modified to allow continuous simulation, while models such as HSPF (Johanson et al., 1984) and STORM (U.S. Army Corps of Engineers, 1977) were originally developed for continuous simulation.

Along a parallel path, the evolution of derived probability distribution models has occurred. This form of continuous analysis uses probability distributions of rainfall volume, duration, interevent time and intensity. Through simplifications in catchment hydrology and system hydraulics, closed-form mathematical solutions can be derived for a variety of quantity and quality control performance measures. Fewer input parameters and the efficiency of computation allow for a broad range of design alternatives to be analyzed quickly. Contributions to derived probability distribution methods are numerous (e.g., Adams and Bontje, 1984; Chan and Bras, 1979; Howard, 1976; Loganathan and Delleur, 1984).

While event modeling is practical in many situations, the evolution of drainage issues has rendered it less appropriate. The primary concerns with this approach include: (i) failure to address quality control issues, (ii) poor handling of antecedent conditions, (iii) unpredictable response to less frequent events, and (iv) uncertainty in catchment parameters. Funds for monitoring runoff response have diminished, so many developing urban catchments are ungauged; therefore, the data necessary to calibrate the event models is not available.

The shift to combined quantity and quality control design has introduced new design criteria. While runoff volume is still of importance, other characteristics such as number of system overflows, volume of overflows and fraction of runoff and pollution controlled are more appropriate terms for measuring quality control. To investigate these measures, long term rainfall records are used in continuous analysis methods. Most models are capable of predicting these control measures, otherwise they require updating or risk being left behind.

10.1.1 Study Objectives

Extensive research has been conducted on the development of derived probability distribution models. The results from these models have been favourable. Due to a lack of measured field data there has been a requirement to validate or compare analytical results to other continuous analysis approaches, most often SWMM and STORM. These comparisons have generally supported the validity of these models. The basis for model comparison has been runoff

quantity control measures. Using long-term rainfall data, revised performance criteria can be quantified including runoff volume and number of events and spill volume and number of events. These measures provide an appropriate picture of drainage system performance.

The application of analytical models in parallel with continuous simulation models requires further study. The purpose of this study is to outline a continuous analysis methodology to provide consistent results between models. Preliminary analyses can be performed using analytical modeling, which is time efficient for planning purposes. More detailed analyses are undertaken using continuous simulation modeling, which is accurate for design purposes. This conjunctive use of models requires calibration of the analytical results to the continuous simulation results.

A sensitivity analysis was performed on the model input parameters using factorial experimental design methods. This identified the input parameters generating the greatest response in the runoff quantity control measures. These parameters were then employed in the calibration exercise. By varying these parameters, the characteristic response of several runoff quantity control measures was observed. Comparison of the response curves from each model provided an opportunity to determine a suitable calibration approach. Several test scenarios were run to verify the calibration curves and the compatibility of the two modeling approaches. Some additional concerns about the response of each model were identified.

10.2 Analysis by Continuous Simulation

10.2.1 StormWater Management Model (SWMM)

SWMM is a widely used stormwater modeling tool and is well accepted by water resources professionals. For the present study, it was selected as an appropriate model for comparison with the analytical models. Previous studies, including Adams (1996), have used continuous simulation to generate data for a variety of quantity and quality control results. Past comparisons have been favourable, but a better compatibility between the modeling approaches is required to use the models interchangeably.

SWMM version 4.31 generated the continuous simulation results based on a sample urban catchment of 10 ha. Since the focus was on runoff quantity control measures, only the RAIN and RUNOFF blocks were employed. An external program, MTOPOND, was used to generate data on numbers of spills and spill volumes.

RUNOFF Block Input Data

The RUNOFF block input file requires an extensive understanding of the catchment hydrology and hydraulic configuration. As a generic study, attempts were made to ensure that all inputs were consistent with an average residential or mixed-use community. Previous studies (Adams (1996), Hollingworth (1996), Huber and Dickinson (1988) and Viessman et al. (1989)) were used to confirm appropriate input values for the simulations.

Meteorological Data

Local hourly rainfall data from Toronto Pearson International Airport (AES Station 6158655) for the period 1960 to 1992 were input to the RUNOFF block. This information did not include snowfall, thus the winter months (December through February) were not simulated.

Monthly evaporation data were acquired from the Canadian Climate Normals for Hamilton, Ontario covering the years 1955-1980. Table 10.1 outlines the data input to the SWMM model.

Table 10.1 Hamilton monthly evaporation data (mm).

Jan	Feb	Mar	Apr	May	June
0.0	0.0	0.0	87.9	111.0	124.8
July	Aug	Sept	Oct	Nov	Dec
143.5	123.3	81.3	46.2	0.0	0.0

Constant Catchment Characteristics

Since the input to the RUNOFF block is extensive, some parameters not directly related to runoff control were fixed as constants. These parameters include catchment area, slope and width, Mannings n and percent of imperviousness with zero depression storage (PCTZER). The values are outlined in Table 10.2.

Table 10.2 Catchment characteristics.

Area	10 ha
Slope	1.0%
Impervious Mannings n	0.013
Pervious Mannings n	0.25
Catchment Width	400 m
PCTZER	0%

Variable Catchment Characteristics

Several catchment characteristics which directly impact runoff volume and runoff quantity control measures were varied during the model analyses. Specifically, these included: percent imperviousness (%IMP), impervious and pervious depression storage (IDS & PDS) and Horton's infiltration parameters. Infiltration calculations in SWMM require the maximum infiltration rate (WILMAX), the minimum infiltration rate (WILMIN), the decay rate of infiltration (DECAY) and a regeneration constant (REGEN). These parameters were expected to have the greatest impact on runoff generation. Specific values used in the analysis are discussed in the factorial design and calibration sections.

10.2.2 MTOPOND

The MTOPOND model is a continuous simulation model developed to predict both quantity and quality control performance of stormwater systems utilizing storage. Both wet and dry pond configurations can be modeled. This study focused on quantity control performance of extended detention dry ponds, thus no permanent storage was included in the simulations. MTOPOND is a FORTRAN program written by Graham Bryant based on the POND model (MOEE, 1994) and was developed for a study contracted by the Ontario Ministry of Transportation (Adams, 1996).

The MTOPOND model reads in data generated by the SWMM RUNOFF block. The program uses a mass balance approach to determine flow through and bypassing a storage facility. If the pond is full and the inflow rate is greater than the outflow, additional runoff will by-pass the pond. The input parameters impacting runoff quantity performance in extended detention dry ponds are outlined in Table 10.3.

Pond outflow at each time step is based on an SCS triangular hydrograph (maximum outflow is equal to twice the average flow) and the pond drawdown time. The outflow rate is calculated at each time step with a minimum flowrate equal to 5% of the average flowrate, thus allowing for complete drainage of the facility.

Table 10.3 MTOPOND parameter values.

Smallest Active Storage Volume	500 m ³ (5 mm)
Largest Active Storage Volume	1500 m ³ (15 mm)
Active Storage Depth	1 m
Permanent Pond Volume	0 m ³ (0 mm)
Data Timestep	60 min
Calculation Timestep	300 s
Retention Time	24 hr or 48 hr
Maximum Inlet Capacity	1 m ³ /s

The quantity control performance outputs from the program include: total runoff volume (m^3), total overflow volume (m^3) and the number of spills for the entire record. These values are reduced to average annual depths using the catchment area.

10.3 Analysis By Analytical Probabilistic Models

10.3.1 Background

Analytical probabilistic modeling provides an alternative approach to the analysis of urban drainage systems. The primary benefit of the analytical models lies in the availability of explicit mathematical solutions to performance measures. A range of system design elements or required performance levels can be investigated with ease while incorporating the full range of meteorological conditions. The analytical models are developed with derived probability distribution theory. Probability density functions (PDFs) describing the input meteorology to the system are transformed to PDFs of system performance parameters by relationships describing the system hydrology and hydraulics.

These models allow for the efficient analysis of stormwater systems using relatively few input parameters. Although these models are approximate, they are useful for planning purposes and preliminary investigations. The model evolution began with derivations for runoff quantity control measures. As the emphasis in stormwater management has shifted to include quality, additional runoff quality control derivations have been introduced. Prior to formalizing the quality analysis components of the analytical probabilistic models, an in-depth study of the relationship between the quantity analysis components of these models and simulation models was undertaken and their calibration was explored in the present work.

10.3.2 The Urban Drainage Model

The development of analytical models is well documented. Early work on these developments was done by Howard (1976), Smith (1980) and Adams and co-workers (e.g., Adams and Bontje, 1984). The model derivations are given in Adams and Bontje (1984) and are not presented in detail herein.

Incorporating simplifying assumptions, Adams and Bontje (1984) developed the analytical models using derived probability distribution theory. Figure 10.1 illustrates the nature and components of the drainage system used in the derivation.

Figure 10.1 Urban drainage system schematic (Adams and Bontje, 1984).

To develop the expressions for runoff quantity control, the input precipitation (v) is transformed into runoff (v_r) using the following relationship:

$$v_r = \begin{cases} 0 & ; v \leq S_d \\ \phi(v - S_d) & ; v > S_d \end{cases} \quad (10.1)$$

where:

- v_r = runoff volume (mm)
- v = rainfall volume (mm)
- S_d = depression storage on the catchment expressed as an equivalent uniform depth across the entire catchment (mm)
- ϕ = runoff coefficient

A catchment reacts to rainfall by first filling the depression storage volume, S_d , prior to runoff generation. The rainfall volume exceeding depression storage, $(v - S_d)$, is multiplied by the runoff coefficient, ϕ , to determine the runoff volume, v_r . The total losses, S_d and $(1 - \phi)(v - S_d)$ can be viewed as a combination of infiltration and some evaporation losses. Assuming that depression storage is empty at the start of each rainfall event accounts for most of the evaporation volume.

Using exponential PDFs of the rainfall characteristics and the expressions for runoff volume, Adams and Bontje (1984) developed key equations describing runoff quantity control measures. These are used in the sensitivity and calibration exercises. The equations presented below assume that the storage facility is full at the end of the previous runoff event. As a result, the analytical models are conservative in the calculation of the runoff quantity control measures.

In the analytical models, the number of average annual runoff events, n_R (/yr), outlines the number of rainfall events which actually generate surface runoff to be routed through a stormwater drainage system and is described by

$$n_R = \theta e^{-\zeta S_d} \quad (10.2)$$

where θ = average annual number of rainfall events
 ζ = inverse of the mean rainfall event volume (1/mm)

The average annual surface runoff volume washing off the catchment, R (mm/yr), is described by:

$$R = \theta \frac{\phi}{\zeta} e^{-\zeta S_d} \quad (10.3)$$

To calculate two additional runoff quantity control measures (i.e., average annual number of spill events and average annual spill volume), the probability per event of a spill of any magnitude is required as described by:

$$G_p(0) = \left[\frac{\lambda/\Omega}{\lambda/\Omega + \zeta/\phi} \right] \left[\frac{\psi/\Omega + \zeta/\phi e^{-(\psi/\Omega + \zeta/\phi) S_a}}{\psi/\Omega + \zeta/\phi} \right] e^{-\zeta S_d} \quad (10.4)$$

where:

λ = inverse of the mean rainfall event duration (1/hr)
 ψ = inverse of the mean interevent time (1/hr)
 Ω = controlled release rate from storage (mm/hr)
 S_a = active storage volume averaged over catchment area (mm)
 $G_p(0)$ = probability per rainfall event of any spill of any magnitude

For each runoff event, there is an opportunity for the drainage system capacity, including storage, to be exceeded. When this occurs a spill event is recorded. The average annual number of spill events, n_s (/yr), is described by:

$$n_s = \theta G_p(0) \quad (10.5)$$

For each spill event, there is an associated volume of runoff which is spilled untreated into a receiving system. The average annual spill volume, P_u (mm/yr), is described by:

$$P_u = \theta \frac{\phi}{\zeta} G_p(0) \quad (10.6)$$

These latter five equations form the basis for the analysis of runoff quantity control measures using the analytical models.

10.3.3 Input Parameters

Implementing the analytical probabilistic models requires significantly less effort than the continuous simulation approach. Simplifications in catchment hydrology greatly reduce the number of input parameters. Most inputs to the analytical models are combinations of several parameters used in the SWMM RUNOFF block. Values used herein are based on previous studies (Adams, 1996).

Meteorological Data

The hourly rainfall record input to the SWMM RAIN block was analyzed to determine the basic rainfall statistics. The data were input to a FORTRAN analysis program, prescribing a 2 hour interevent time definition, with results as follows:

- Inverse of the average annual rainfall volume, $\zeta = 0.196$ 1/mm
- Inverse of the average annual rainfall duration, $\lambda = 0.278$ 1/hr
- Inverse of the average annual interevent time, $\psi = 0.018$ 1/hr
- Average annual number of rainfall events, $\theta = 108.6$ /yr

Catchment Characteristics

The runoff coefficient (ϕ), active storage volume (S_a), controlled outflow rate (Ω) and depression storage (S_d) must be quantified based on the system configuration and catchment hydrology.

The depression storage value represents a composite amount of rainfall depth that is abstracted from both impervious and pervious areas. This is unlike SWMM which requires the specification of a separate depression storage value for pervious and impervious areas.

The runoff coefficient does not represent imperviousness as its counterpart, %IMP, does in SWMM. The value of ϕ indicates the percentage or fraction of rainfall escaping depression storage which is infiltrated or evaporated.

The active storage volume is determined by the size of the storage pond and describes the catchment area depth available to control runoff as:

$$S_a = \frac{V_a}{A_c} 10^3 \quad (10.7)$$

where:

$$\begin{aligned} V_a &= \text{active storage volume at full depth (m}^3\text{)} \\ A_c &= \text{catchment area (m}^2\text{)} \end{aligned}$$

The active storage volume combined with the design detention time of the facility define the controlled outflow rate as:

$$\Omega = \frac{S_a}{t_d} \quad (10.8)$$

where:

t_d = average retention time of the pond (hr)

The factorial design and calibration exercises incorporate a range of values of the parameters described herein. Specific values are presented in the experimental factorial design and the calibration exercise sections.

10.4 Factorial Experimental Design

10.4.1 Theory

In stormwater management modeling, there are many parameters which influence the response of drainage systems. Model input parameters are usually treated equally, yet several key parameters often impact the results more significantly than others. Using factorial experimental design, the effects of changing input parameter values on continuous simulation and analytical model results can be illuminated.

Factorial experimental design involves selecting a fixed number of parameters (factors) for analysis and a fixed number of typical values (levels) for each factor. Experiments are then conducted with all combinations of these parameters and their values. The process is described as a $l_1 \times l_2 \times \dots \times l_k$ factorial design, where there are l_1 levels for the first parameter, l_2 levels for the second and so on. For example, a $4 \times 2 \times 2$ factorial design requires $4 \times 2 \times 2 = 16$ experimental runs.

Usually, only two level factorial designs are analyzed where each parameter is assigned one high and one low value. Box et al. (1978) outlines the importance of these designs:

1. Generally, two-level factorial designs require few runs, while revealing major trends. Even though a full range of values is not explored, the analysis can direct the process towards further, refined experimentation.
2. Two-level fractional factorial designs are excellent for performing preliminary investigations. By looking at all parameters on a general level, the more crucial components can be analyzed in detail later on.
3. These designs can be used in steps to develop a final design which matches the complexity of the initial problem.
4. The interpretation of the designs is essentially common sense and can be attained through simple arithmetic.

Calculation of Main Effects

The primary objective of factorial experimental design is to quantify the main effect of a parameter. Main effects are defined as the impact that changing a parameter value between levels has on a prescribed result, while holding other parameters constant. Effects are calculated for each factor, and the value of the effect describes the relative sensitivity of a result to altering the parameter between two levels. The greater the value of the effect, the higher the impact of that factor on an analysis. The sign of the effect reflects a net increase or decrease in the result.

Calculation of Secondary Effects or Two-Factor Interactions

Main effects illustrate the impact that a single parameter has on an output, but it does not reveal whether two parameters together exhibit an additive effect. These secondary (interaction) effects are calculated using the difference between the average effect of one factor at a low value and at a high value of the second factor.

Higher levels of interactions, such as three-factor interactions, can be quantified, but are generally less significant. The greater the number of parameters, the less likely all of the parameters will interact, driving the value of the interaction effect lower.

Fractional Factorial Designs

The required number of experimental runs increases geometrically with the number of factors to analyze. Under such circumstances, it becomes possible to extract estimates of the effects by performing only a fraction of the full factorial design. This results in large time savings and helps focus the analysis on the parameters generating the greatest response in the performance measures.

In factorial analysis, main effects are largest, decreasing in magnitude through two-factor interactions, three-factor interactions and higher level interactions. When the number of parameters introduced into a study is high, a select few may have no distinguishable effects at all. Under these conditions there tends to be a redundancy in terms of an excess number of interactions that can be estimated and possibly an excess number of parameters being studied. By removing levels of testing, the ability to calculate the higher interactions is lost, but main effect values are changed only slightly. As more information about specific parameters and secondary interactions is required, additional fractions can be run to improve the accuracy of the effects.

Box et al. (1978) explore full and fractional factorial designs in detail and can be referenced for additional information.

10.4.2 Continuous Simulation Factorial Design

Understanding continuous simulation model sensitivity required isolating key input parameters and their effects on a select number of runoff quantity control performance measures. The continuous simulation investigation in this study focused on the following five measures:

- average annual volume of runoff, R
- average annual volume of infiltration, I
- average annual volume of evaporation, E
- average annual volume of spill, P_u
- average annual number of spill events, n_s

The key continuous simulation input parameters are contained in the RUNOFF block input file. The parameters chosen as most likely having a significant impact on the above five performance measures, and the ranges chosen for the factorial analysis, are illustrated in Tables 10.4a and 10.4b. Refer to the list of notation to clarify the definitions of terms used in these analyses.

The values used in this analysis have been extrapolated from a variety of resources, including Adams (1996), Hollingworth (1996), Viessman et al. (1989) and Huber and Dickinson (1988). The numbers represent a reasonable range of design values derived from urban stormwater modeling practice.

Table 10.4a Continuous simulation model factorial design factors and values for imperviousness and depression storage.

	%IMP	IDS (mm)	PDS (mm)
Low	30	0.5	0.5
High	70	2.0	4.5

Table 10.4b Continuous simulation model factorial design factors and values for the Horton model.

	WILMAX (mm/h)	WILMIN (mm/h)	DECAY (s ⁻¹)	REGEN (d)
Low	25	3.75	0.00056	3
High	125	11.50	0.00115	7

A 2⁷ factorial design (128 simulation runs) was necessary to analyze all combinations of factors and values. Each continuous simulation required approximately 45 minutes; thus, a considerable analysis time was needed.

Fractional factorial designs provide an appropriate estimate of the main effects; therefore, for this experimental design, a 2_{III}^{7-4} design (eight simulation runs) was used.

After running the first fraction, estimates of the main effects were determined, but no secondary effects could be calculated. A second 1/16 fraction of eight additional runs was conducted to provide the additional data. The design matrices for the two analyses are illustrated in Tables 10.5 and 10.6. The '+' symbol represents the substitution of the appropriate 'High' value from Table 10.4, and the '-' symbol indicates the substitution of the 'Low' value.

Table 10.5 First 1/16 fractional design for continuous simulation analysis.

	%IMP	IDS	PDS	WILMAX	WILMIN	DECAY	REGEN
		(mm)	(mm)	(mm/h)	(mm/h)	(s ⁻¹)	(d)
Run #	1	2	3	4	5	6	7
1	-	-	-	+	+	+	-
2	+	-	-	-	-	+	+
3	-	+	-	-	+	-	+
4	+	+	-	+	-	-	-
5	-	-	+	+	-	-	+
6	+	-	+	-	+	-	-
7	-	+	+	-	-	+	-
8	+	+	+	+	+	+	+

Table 10.6 Second 1/16 fractional design for continuous simulation analysis.

	%IMP	IDS	PDS	WILMAX	WILMIN	DECAY	REGEN
		(mm)	(mm)	(mm/h)	(mm/h)	(s ⁻¹)	(d)
Run #	1	2	3	4	5	6	7
9	+	+	+	-	-	-	+
10	-	+	+	+	+	-	-
11	+	-	+	+	-	+	-
12	-	-	+	-	+	+	+
13	+	+	-	-	+	+	-
14	-	+	-	+	-	+	+
15	+	-	-	+	+	-	+
16	-	-	-	-	-	-	-

10.4.3 Results of the Continuous Simulation Investigation

The resulting values of the main effects are presented in Table 10.7, and the secondary effects are listed in Table 10.8. For example, the average volume of runoff that was generated by the 16 experimental runs is 255 mm/yr. The main effect of percent imperviousness was 196 mm/yr. By changing the value of %IMP from 30% to 70%, the volume of runoff increased on average by 196 mm/yr. Similarly, changing the impervious depression storage (IDS) from 0.5 mm to 2.0 mm decreased the volume of runoff by 24 mm/yr.

Table 10.7 Main effects for continuous simulation investigation.

Type	Units	% IMP	IDS	PDS	WIL- MAX	WIL- MIN	DE- CAY	RE- GEN	AVG. VALUE
		1	2	3	4	5	6	7	
R	mm/yr	196	-24	-3	-7	-9	2	1	255
I	mm/yr	-217	0	3	6	8	-2	-1	270
E	mm/yr	30	23	0	0	-1	0	0	36
n_s	/yr	1.9	-0.1	-0.2	-0.3	-0.3	0.1	0.1	1.4
P_u	mm/yr	16.1	-1.0	-1.3	-3.7	-4.1	1.4	1.3	12.2

Table 10.8 Selected secondary effects for continuous simulation investigation.

Type	Units	%IMP x IDS	%IMP x PDS	%IMP x WILMAX	%IMP x WILMIN	%IMP x DECAY	%IMP x REGEN
		1,2	1,3	1,4	1,5	1,6	1,7
R	mm/yr	6	-1	0	-2	0	-2
I	mm/yr	-1	1	0	2	0	2
E	mm/yr	-4	0	0	0	0	0
n_s	/yr	0.1	0	0	0	0	-0.1
P_u	mm/yr	0.7	-0.3	0.7	-0.1	-0.4	-1.0

The values in the analysis were obtained as follows:

- Values for runoff, infiltration and evaporation were taken from the RUNOFF block output file and area-averaged based on the 10 ha catchment.
- Values for n_s and P_u were extracted from the MTOPOND summary file. These values represent a dry pond scenario with a 15 mm active storage capacity and an average detention time of 24 hours.

Effect of Percent Imperviousness

The most pronounced impacts were produced by the percent imperviousness, and the consequences of these results must be explored. No other single input parameter had such a significant impact on the runoff quantity control results. This analysis suggests that continuous simulation results may be excessively dependent on this parameter.

The results identified that the runoff quantity control measures are most sensitive to %IMP. In the analysis of average annual infiltration volume, none of the four Horton parameters included in this analysis had near the impact of percent imperviousness. The impervious depression storage played a role in evaporation, because IDS directly impacts the rainfall volume available for evaporation. Across the range of runoff quantity control measures, depression storage and infiltration rates demonstrated little impact on the results over the range of parameter values chosen.

The impact of percent imperviousness values on model results must be made aware to modeling practitioners. Percent imperviousness must be quantified carefully in the use of continuous simulation modeling. Impervious areas must be accurately measured, including separating areas which are directly connected and those which discharge to pervious areas. More generally, percent imperviousness is critical in any hydrologic modeling, continuous or event based.

Effect of Depression Storage

Results from Tables 10.7 and 10.8 suggest that the values for impervious and pervious depression storage had little impact on the runoff quantity control results. The impervious area dominates the results; therefore, pervious depression storage was ineffective for influencing the model results. Impervious depression storage impacted the evaporation results and to a lesser extent runoff volume. The effect on evaporation was expected, because impervious depression storage generated the majority of the evaporation loss, while losses from pervious depression storage were governed by infiltration.

Effect of Horton's Equation Infiltration Parameters

Even though a broad range of infiltration parameter values were used in this analysis, it appears that specifying average input values may yield similar results. This analysis questions the need to research detailed soil profiles and characteristics. The inputs for a clayey soil differs greatly from a sandy soil, but the ultimate continuous simulation result may not vary significantly. Searching for accurate infiltration and regeneration rates may be premature and unnecessary, especially at the planning stage.

Based on the results, the impervious factors, %IMP and IDS, had complete dominance over the runoff quantity control measures. It may be inappropriate to accurately quantify a large number of parameters for the RUNOFF block input file, when only a few actually impact the runoff results. Remember, these results were generated using a modest number of parameters over a broad range of input values.

10.4.4 Analytical Model Factorial Design

To attempt calibration with the continuous simulation approach, it was necessary to perform a similar experimental design and analysis for the analytical models. The factorial design was directly linked to the runoff quantity control expressions in Equations 10.2 through 10.6, and the performance measures of concern in the analytical model include

- Average annual volume of runoff, R
- Average annual number of runoff events, n_R
- Average annual volume of spill, P_u
- Average annual number of spill events, n_s

In this analysis, the meteorological parameters (ζ , λ , ψ , θ) were fixed based on location, and the values for Ω and S_a were assumed to represent specific design criteria. Only the runoff coefficient (ϕ) and depression storage (S_d) required detailed analysis for their impact on the runoff quantity control measures. The ranges for these factors are presented in Table 10.9.

Table 10.9 Analytical model factorial design factors and values.

	ϕ	S_d (mm)
Low	0.30	0.5
High	0.70	4.5

Table 10.10 Factorial design for analytical model.

	%IMP	IDS (mm)
Run #	1	2
1	-	-
2	+	-
3	-	+
4	+	+

The analytical models were analyzed using a 2^2 factorial design, and because the results could be calculated analytically in four experimental runs, the task was simple. Similar to the continuous simulation factorial design, the analysis strategy is presented in Table 10.10.

10.4.5 Results of the Analytical Model Investigation

The full factorial analysis was accomplished using a spreadsheet and simple arithmetic. The numerical results are presented in Tables 10.11 and 10.12. The average values of the performance measures were included to illustrate sensitivity. The analysis was based on the meteorological parameters listed in Section 10.3.3 and an extended detention dry pond facility with 15 mm of active storage capacity and an average detention time of 24 hours.

Table 10.11 Main effects for analytical model investigation.

Type	Units	ϕ	S_d	AVG. VALUE
		1	2	
R	mm/yr	146	-136	183
n_R	/yr	0	-54	72
n_s	/yr	3.3	-2.1	2.9
P_u	mm/yr	14.2	-6.7	9.0

Table 10.12 Secondary effects for analytical model investigation.

Type	Units	$\phi \times S_d$
		1,2
R	mm/yr	55
n_R	/yr	0
n_s	/yr	1.2
P_u	mm/yr	5.3

Effect of the Runoff Coefficient

Table 10.11 illustrates that the runoff coefficient had a substantial impact on the runoff quantity control results. This analysis confirms that the runoff coefficient value should not be assumed or simply estimated. Fortunately, most practitioners are aware of the importance of selecting appropriate values of runoff coefficients. The significant magnitude of the secondary effect, between runoff

coefficient and depression storage, on runoff volume suggests factor interaction. Some comfort in this observation may be derived from the fact that the effect of the runoff coefficient and depression storage value played a complimentary role in the analysis of performance measures.

Effect of Depression Storage

In contrast to the continuous simulation model analysis, depression storage in the analytical model analysis impacted the results significantly. The relationship between depression storage and runoff coefficient in the analytical models suggests more realistic catchment runoff generation. The depression storage did not exceed the runoff coefficient effect in any category (except number of runoff events), but it appeared to restrict the impact of the runoff coefficient.

A question arose in the calculation of average annual number of runoff events. As stated in Equation 10.2, the only parameter affecting n_R was depression storage. This raised concern about assumptions of the rainfall-runoff relationship in the analytical models. Intuition suggested that the runoff coefficient should impact this result. From a broader perspective, this point may be inconsequential. The number of spill or runoff events is usually less significant in an analysis than the volume of runoff or spills. The number of events fails to indicate the relative magnitude of such occurrences which is likely to have a greater impact on an overall design concept.

The analytical models stress simplicity. A practitioner is only required to quantify values for ϕ and S_d . Because there are only two catchment parameters to specify, their impact on the analysis is greatly increased. The decision to use either the continuous simulation or analytical models is a compromise in complexity. The context in which the results are used must be kept forefront in the modeler's mind.

10.5 Calibration of the Analytical Model Results

10.5.1 Development of a Calibration Strategy

The identification of sensitivities in each model provided useful insight into the response of the continuous simulation and analytical models for runoff quantity control measures. An appropriate calibration approach should incorporate the parameters from the previous analysis that had the most significant impact on the performance results. The purpose of this calibration exercise was to develop a modeling tool providing a link between the continuous simulation and the analytical models. The task was to calibrate results for two major runoff quantity control measures: average annual number of spills (n_s) and average

annual volume of spills (P_u) from the analytical model to the continuous simulation results. Using the calibration, values for average annual volume of runoff (R) and average annual number of runoff events (n_R) could be compared for verification.

The factorial analysis of continuous simulation model results suggested that percent imperviousness generated the greatest response in the runoff quantity control results. The impact of the other six were insignificant; therefore, these factors were fixed in the calibration exercise and set to average amounts. These values, outlined in Table 10.13, are taken from previous studies (Adams (1996), Wisner (1989)) involving analysis of urban drainage systems in Ontario. The percent imperviousness was varied from 0% to 100% to illustrate its impact.

Table 10.13 Fixed values for RUNOFF Block input parameters.

Impervious Depression Storage	1.5 mm
Pervious Depression Storage	4.5 mm
Maximum Infiltration Rate	50.0 mm/h
Minimum Infiltration Rate	7.5 mm/h
Soil Decay Rate	0.00056 s ⁻¹
Soil Regeneration Rate	0.00142

Factorial analysis of the probabilistic models suggested that both ϕ and S_d should be involved in the calibration exercise. The design parameters, S_a and Ω , also became a part of the comparison process.

To approach the calibration exercise, the runoff quantity control measures of interest were plotted against the percent imperviousness and runoff coefficient (assuming that %IMP = 50% is equal to $\phi = 0.50$ for the purposes of this comparison). Testing various combinations of storage and detention time, which together determined Ω using Equation 10.8, the initial correspondence of results without calibration was developed.

Figures 10.2 and 10.3 illustrate two such graphs for n_s and P_u . The values chosen to represent typical active storage volumes for an urban catchment ranged between 5 mm and 15 mm (or 500 m³ and 1500 m³ for the 10 ha catchment). Detention times of 24 hours and 48 hours were used because they represented suspended solids removal rates associated with common runoff quality control objectives. Values greater than 48 hours typically increased the probability of another storm event occurring and increased spill numbers at a greater rate than the suspended solids removal efficiency was improved.

Figure 10.2 Analysis of n_s over a range of S_d for continuous simulation and analytical models ($S_a = 7$ mm, $t_d = 24$ h).

Figure 10.3 Analysis of P_u over a range of S_d for continuous simulation and analytical models ($S_a = 11$ mm, $t_d = 48$ h).

Figures 10.2 and 10.3, and other graphs not shown, illustrated that the two models, uncalibrated, produced similar results. The analytical model results agreed with the continuous simulation model results, provided the correct depression storage value was used. It appeared that separate S_d values were required to calibrate the n_s and P_u performance measures.

The plotted results suggested that the following two strategies for calibration may be appropriate:

- Generate a shift term, ϕ_s , to convert the SWMM factor %IMP/100 into a calibrated runoff coefficient, ϕ , for the analytical models. Depression storage, S_d , could be represented as an average of the impervious and pervious depression storage used in the continuous simulation model.
- Set the runoff coefficient, ϕ , equal to %IMP/100, and calibrate the depression storage value, S_d , which best fits the continuous simulation results.

From a detailed analysis of each method, the simplest and most accurate approach was found to be calibration to depression storage. The following sections illustrate the steps required to derive the calibration curves.

10.5.2 Calibration of the Average Annual Number of Spills

A large amount of data was required to calibrate the analytical values of n_s . Continuous simulation results for n_s and P_u were generated for %IMP from 20% to 80%. Results for each runoff quantity control measure were produced at all combinations of active storage and detention time.

An Excel spreadsheet was set up to optimize the fit of n_s for the analytical model results to the continuous simulation model results. The analytical expressions, Equations 10.4 to 10.6, were entered over the range of active storage volumes. For each situation, the input parameters and the continuous simulation model results were also entered. The value of S_d was iterated by the Excel Solver to produce a best fit with the continuous simulation data as judged by the R^2 value calculated as:

$$R^2 = 1 - \frac{\sum (y_a - y_c)^2}{\sum (y_a - \ln(\bar{y}_a))^2} \quad (10.9)$$

where:

- y_a = analytical model results at a given storage volume
- y_c = continuous simulation model result at a given storage volume
- $\bar{y}_a$ = average of all analytical model results in the storage volume range

The maximum value of R^2 was deemed to provide the best fit. The depression storage values providing the best calibration to the continuous simulation model results for n_s are listed in Table 10.14.

Table 10.14 Depression Storage Results for the Calibration of n_s .

Active Storage, S_a (mm)	Depression Storage, S_d (mm)	
	24 h Detention	48 h Detention
5	3.81	6.30
7	3.65	6.05
9	4.01	6.24
11	4.58	6.75
13	5.00	7.38
15	5.57	7.85

Performing separate optimizations for each detention time appeared to be appropriate. To establish a calibration curve, the results were plotted in Figure 10.4 and regressed using the Excel Trendline function.

Figure 10.4 Calibration curves for n_s using a second order polynomial regression.

Both a linear and second order polynomial regression were performed on the data. The second order polynomial was found to produce a better fit and the corresponding equations were:

$$\bullet \text{ 24 hour detention: } S_d = 0.0176S_a^2 - 0.1597S_a + 4.0706 \quad (10.10) \\ (R^2 = 0.98)$$

$$\bullet \text{ 48 hour detention: } S_d = 0.0237S_a^2 - 0.2985S_a + 7.1033 \quad (10.11) \\ (R^2 = 0.97)$$

Although a second order polynomial was more complicated than the linear regression, it was still appropriate over the analyzed range of active storage. Regression over the range of 9 mm to 15 mm, resulted in a more linear relationship, but further testing would be required to verify this trend beyond 15 mm.

One encouraging observation was that the difference in calibrated S_d values over the range of detention times was almost constant. A base calibration for S_d at a low detention time and shift terms for higher detention times is available for future investigation.

The values of S_d for 48 hour detention were very high. They ranged from 6.0 mm to 8.0 mm and had no practical interpretation. This indicated that the analytical model was generating a large number of small, insignificant spill events which raised the calibrated S_d value. This may result from assuming that the storage volume was full at the end of the last event. The validity of Equations 10.10 and 10.11 was investigated through verification.

10.5.3 Calibration of the Average Annual Spill Volume

Using a different spreadsheet and the same continuous simulation model results, values for annual spill volume were subjected to the same analysis procedure. The value of S_d was iterated to produce a best fit to the continuous simulation model results. Calibrating over the range of 20% to 80% imperviousness, the resulting S_d values are given in Table 10.15

The calibration curves plotted in Figure 10.5 illustrate that the relationships generated for P_u were different than those of n_s . The curves did not have a local minimum, but become asymptotic to the x-axis as the active storage volume was decreased below 5 mm. The curves for 24 and 48 hour detention times were not parallel and diverged as active storage volume increased. Under these circumstances, a power equation provided the best fit, although a linear regression was reasonable as well.

Table 10.15 Depression storage results for calibration of P_u

Active Storage, S_a (mm)	Depression Storage, S_d (mm)	
	24 h Detention	48 h Detention
5	0.06	0.47
7	0.11	0.75
9	0.27	1.18
11	0.47	1.68
13	0.72	2.21
15	1.05	2.78

Figure 10.5 Calibration curves for P_u using a power regression.

The corresponding equations for best fit were

- 24 hour detention: $S_d = 0.0006S_a^{2.7662}$ (10.12)
($R^2 = 0.99$)

- 48 hour detention: $S_d = 0.0315S_a^{1.6534}$ (10.13)
($R^2 = 1.00$)

The fit of the regression equations was excellent, and the resulting values of S_d were physically reasonable for an urban catchment. The ability to calibrate P_u was more desirable, because the volume of spills could be converted to mass of pollutants using representative concentrations. This provides estimates of pollution in an aquatic environment.

10.5.4 Summarizing the Calibration Results

Calibration between the continuous simulation and analytical models was not simple. The differences in the assumptions of each model and input parameters complicated the procedure. The calibration curves for n_s and P_u , although different, indicated that smooth functions can be used to calibrate the models. The n_s curves remained parallel for each detention time, but the S_d values were unrealistically high. The P_u curves separated with increased active storage but provide more acceptable values of depression storage.

10.6 Verification of the Calibration Procedure

The calibration in Section 10.5 was performed using average catchment parameters. Only infrequently would such design conditions appear in practice. Three scenarios were chosen to ensure that the calibration curves were applicable for general design.

Results of the sensitivity analysis suggested that changing the values of impervious and pervious depression storage had little impact on the runoff quantity control results. The rainfall, evaporation and soil infiltration values were maintained at constant levels. The major impacts tested include:

- varying the value of %IMP across the range of 20% to 80%;
- verifying the insensitivity of the control measures to depression storage; and
- testing a range of active storage volumes and detention times

Details of three test scenarios are outlined in Table 10.16.

Table 10.16 Continuous simulation input parameter values for numerical testing.

Scenario	%IMP	IDS (mm)	PDS (mm)	S_a (mm)	t_d (h)
Commercial					
Complex	75	0.50	1.50	10, 12, 14	24
Residential					
Housing	40	1.00	3.50	5, 7, 9	48
Urban					
Park Site	20	0.25	5.00	5, 7, 9	24

Table 10.17 summarizes the results using the calibration equations and the corresponding parameter values input to the continuous simulation and analytical models. The values of n_R and R are assembled in Table 10.18 as additional verification. The S_d value used to calculate the analytical results for n_R and R was extracted from the P_u calibration equation, based on the middle value of S_a from Table 10.16.

Calibration results were excellent for number of spills and spill volume. The difference in number of spills between the continuous simulation and analytical model was less than one, and spill volume was approximately within 3 mm for all design examples. For a planning level analysis, this represented a promising approximation of the continuous simulation results by the analytical models. One remaining concern is the increase of percent difference as the percent imperviousness is reduced.

Table 10.17 Verification results for n_s and P_u .

Scenario	S_a (mm)	n_s (/yr)			P_u (mm/yr)		
		Cont. Simul'n	Analyt'l Model	% Diff.	Cont. Simul'n	Analyt'l Model	% Diff.
Commercial	10	6.9	6.2	10	54.3	51.0	6
Complex	12	4.8	4.3	10	39.7	36.9	7
	14	3.6	3.0	20	29.4	27.2	7
Residential	5	8.4	7.8	7	48.1	49.4	3
Housing	7	4.8	5.5	15	29.1	32.1	10
	9	3.1	4.0	29	18.9	22.2	17
Urban	5	1.6	2.6	63	8.5	5.4	36
Park Site	7	0.9	1.5	67	5.8	3.2	45
	9	0.6	1.0	40	4.1	2.1	49

Table 10.18 Verification results for n_R and R .

Scenario	n_R (/yr)			R (mm/yr)		
	Cont. Simul'n	Analyt'l Model	% Diff.	Cont. Simul'n	Analyt'l Model	% Diff.
Commercial Complex	93.0	96.9	12	394.7	370.5	6
Residential Housing	101.0	93.1	8	206.5	189.8	8
Urban Park Site	97.0	105.8	9	111.3	107.9	3

Computation of the average annual number of runoff events and average annual runoff volume also produced reasonable results. The number of runoff events was a difficult parameter to calibrate given the different assumptions between the two models. The calibration brought the n_R values from the two models to within 10%. Catchment runoff volume was also within the 10% range. Even though the exercise was structured to calibrate n_s and P_u , the n_R and R results were encouraging.

10.7 Conclusions

The objective of this study was to analyze two approaches to stormwater modeling and attempt to calibrate the analytical results to the continuous simulation results for runoff quantity control measures. Each model has advantages and disadvantages. Continuous simulation modeling required more time and detailed input, but is necessary for detailed engineering design. Analytical modeling incorporates many simplifying assumptions which limit the accuracy of the results. As a planning tool, it can be used to run multiple scenarios quickly. Together, analytical modeling and continuous simulation provide the engineer with an efficient and useful suite of stormwater planning and design tools.

This study revealed that continuous simulation is highly sensitive to the specified value of percent imperviousness. This insight is valuable to modelers who must appreciate the need to estimate an accurate value for this parameter. The results do not imply a fundamental flaw in the representation of the catchment hydrology, but highlight the calculation of runoff by SWMM, especially from impervious areas.

Analytical models were not without restrictions. To be used interchangeably with continuous simulation models, assumptions were needed for the relationships between the analytical model runoff coefficient and depression storage values and the continuous simulation model percent imperviousness and impervious and pervious depression storage values. The calibration of one model to another is not as simple as illustrated by the exercise in Section 10.5.

Most modeling practitioners are aware that the percent imperviousness and runoff coefficient are both important in the estimation of runoff, and the results help to reinforce this point. Additionally, this study performs a large amount of comparative analysis between model results. The purpose of the comparisons are ultimately aimed at producing accurate absolute results from both models. Since few field studies have been conducted to quantify runoff quantity control measures, comparative analysis is a preliminary means of analyzing the validity of model results.

This study provides the practicing engineer with an effective approach to stormwater modeling, assuming the modeler understands model input values and their impact on runoff quantity control measures. Analytical models provide a flexible, efficient approach for planning and preliminary design. Continuous simulation can be used for detailed design and analysis after the system design alternatives are screened.

Notation

A_c	Catchment Area, m^2
DECAY	Decay Rate of Infiltration in Horton's Equation, s^{-1}
$G_p(0)$	Probability per Rainfall Event of Any Magnitude Spill
IDS	Impervious Area Depression Storage, mm
n_R	Average Annual Number of Runoff Events
n_s	Average Annual Number of Spill Events
p	Volume of Runoff Spilled from Storage, mm
PCTZER	Percent of Impervious Area with Zero Depression Storage
PDS	Pervious Area Depression Storage, mm
P_u	Average Annual Uncontrolled Spill Volume, mm
R	Average Annual Volume of Runoff, mm
R^2	Goodness of Fit Parameter for Regression Analysis
REGEN	Regeneration of Infiltration Capacity based on Horton Infiltration
S_a	Capacity of Active Storage in Detention Pond, mm
S_d	Depression Storage, mm
t_d	Average Detention Time, h
v	Total Event Rainfall Volume, mm
V_a	Volume of Active Storage at Full Depth, m^3
v_r	Volume of Runoff, mm
WILMAX	Maximum Initial Infiltration Rate, mm/hr
WILMIN	Minimum (Asymptotic) Infiltration Rate, mm/hr
ϕ	Runoff Coefficient
λ	Parameter for Exponential PDF of Rainfall Duration, hr^{-1}
θ	Average Annual Number of Rainfall Events
Ω	Controlled Release Rate from Detention Pond, mm/hr
ψ	Parameter for Exponential PDF of Rainfall Interevent Time, hr^{-1}
ζ	Parameter for Exponential PDF of Rainfall Volume, mm^{-1}
%IMP	Percent Imperviousness of Catchment

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