Regional Estimation of Short-Duration Rainfall Distribution Using Available Daily Rainfall Data.

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The objective of this chapter is to propose methods for estimating the distribution of hourly rainfall based on at-site daily rainfall measurements, and from available regional information. The methods consist of deriving time-resolution independent mathematical models to describe the distribution of rainfalls for different time scales at a single site, and for the whole study region. The proposed models are based on the theory of multifractal multiplicative cascades which describes the transfer mechanism of rain flux from large time scales (e.g. days or longer) to smaller time intervals (e.g. hours or shorter). In the present study, using rainfall data from a network of nine recording raingages in the Montreal region (Quebec, Canada) an empirical investigation is carried out to demonstrate how well rainfall measurements at different time resolutions can be described by such multiplicative cascade processes. Results of the data analysis in this study have provided some credence to this hypothesis since it was found that the probability distributions of rainfalls over a wide range of time scales can be described by one particular theoretically-derived functional form. Further, the good agreement between computed and observed hourly rainfall distributions have indicated the adequacy of the proposed methods. It is expected that a straightforward extension of the method suggested in this study could be used to obtain rainfall distributions appropriate to other time intervals.

Nguyen, V. and H. Wang. 1996. "Regional Estimation of Short-Duration Rainfall Distribution using Available Daily Rainfall Data." *Journal of Water Management Modeling* R191-11. doi: 10.14796/JWMM.R191-11.

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11.1 Introduction

Information on the distribution of rainfall in space and time is critical for various types of hydrologic studies related to the estimation of runoff characteristics. Due to the high temporal and spatial variations of the rainfall process, intensive measurements are usually necessary to obtain reliable information, but they could be impractical and costly. Hence, in practice, rainfall data at the location of interest are often limited (e.g. less than 10 years) or unavailable, or the sampling frequency is not sufficiently high (e.g. more than 1 day). In particular, rainfall intensity data with high temporal resolutions (hours or minutes) are often required as input for runoff simulation models for urban and small rural watersheds, since these areas are generally characterized by fast response. This requirement, however, has frequently become a barrier to the use of these models because short duration rainfall records are often not available for the location considered. On the other hand, daily rainfall data are widely available. This would suggest that if techniques could be developed to derive short time interval (hours or shorter) rainfall characteristics from daily rainfall data available at the site of interest, or from existing data at other sites within a homogeneous region (see, e.g., Nguyen and Chaleeraktrakoon, 1990), these characteristics could be used at locations where only daily rainfall records are available.

Several stochastic models have been developed to describe the rainfall process at a single site (e.g., Waymire and Gupta, 1981; Chang et al., 1984; Entekhabi et al., 1989; Istok and Boersma, 1989). Unfortunately, these models are accurate only for the specific time frame associated with the data used; that is, the inference and deduction based on the proposed models have to be restricted to the particular level of aggregation of the rainfall data from which these models were developed. The accuracy and usefulness of these models are thus rather limited because they cannot describe adequately the rainfall properties at other time scales which are not included in the building of their mathematical structure. For instance, rainfall models based on the Neyman-Scott stochastic point process (see, e.g., Entekhabi et al., 1989; Istok and Boersma, 1989) were found to be inconsistent at different time scales since, for example, if they are fitted to hourly rainfall process, they do not preserve the rainfall characteristics at the daily scale (Foufoula-Georgiou and Guttorp, 1987). Therefore, it was argued that the usefulness of a model should lie in its potential ability to adequately describe the rainfall process at other time scales which are not included in the building of its mathematical structure. It has necessitated the need for formulating models whose mathematical structure will follow the main statistical features of the past history through a continuum of scales. This implies that the suggested model should statistically and simultaneously match various properties of the physical process at different levels of aggregation; whether or not these properties are included in the model. The most important practical implication of such models

is that, from a higher aggregation model we could infer the statistical properties of the process at the finer resolutions which may not have been observed.

Recently, several investigations have found that the multiplicative cascade process is capable of describing the multifractal variability of rainfall over a large range of time and space scales (see, e.g. Schertzer and Lovejoy, 1987; Ladoy et al., 1993). More specifically, the multiplicative cascade process gives rise to scaling statistical moments and scaling probability distribution of rainfalls. In particular, Nguyen and Pandey (1994) have applied this multifractal multiplicative cascades theory to hourly rainfall time series in order to provide some insight into the scaling nature of the probability distribution of the accumulated rainfalls at different time resolutions. On the basis of a scale-independent mathematical model, which describes the probability distributions of rainfalls at various time scales, the distribution of finer time scale rainfalls can be derived from the distribution of daily rainfall data at the site of interest. The present study, a continuation of the previous work (Nguyen and Pandey, 1994), proposes an improved procedure for estimating the distribution of short-duration rainfalls using at-site and regional information of daily rainfall data available in the study region. Results of an illustrative application have indicated the better performance of the proposed method as will be shown in the following.

11.2 Methodology

As mentioned above, geophysical scaling fields, such as a rain field, can be modeled by a multiplicative cascade process, leading to universal multifractals (Schertzer and Lovejoy, 1987). The basic equation of multifractal fields is

$$\Pr(R_\lambda > \lambda^\gamma) \approx \lambda^{-c(\gamma)} \quad (11.1)$$

where:

- $\Pr(.)$ = the probability;
- λ = T/t is the scale ratio, with T being the largest scale and t being the smallest scale of interest;
- R_λ = R/R_t with R_t and R_T being the rainfall intensities at scale t and T , respectively;
- $\gamma > 0$ = the order of singularity, and
- $c(\gamma)$ = the codimension function of the singularity.

Equation (11.1) indicates that the probability distributions of singularities at different resolution scales are related with each other through the scale independent codimension function $c(\gamma)$. The “ $\approx$ ” sign in Equation (11.1) absorbs

the multiplicative part which is a function of γ . Further, it has been shown by Schertzer and Lovejoy (1987) that for continuous cascade process the codimension function $c(\gamma)$ belongs to the universality class and is given by

$$c(\gamma) = \begin{cases} C_1 \left(\frac{\gamma}{C_1 \alpha'} + \frac{1}{\alpha} \right)^{\alpha'}, & \alpha \neq 1 \\ C_1 \exp \left(\frac{\gamma}{C_1} - 1 \right), & \alpha = 1 \end{cases} \quad (11.2)$$

where:

$$1/\alpha + 1/\alpha' = 1,$$

C_1 = the fractal dimension of the singularities contributing to the average values of the field, and

$\alpha(0 < \alpha < 2)$ = the Lévy index indicating to which classes the probability distribution belongs (e.g. the log-normal process corresponds to $\alpha = 2$).

Hence, according to Equation 11.2 only two fundamental parameters C_1 and α are needed to characterize the process considered.

Notice that the basic multifractal equation (Equation 11.1) is not readily applicable in practice, though it is theoretically elegant, because its multiplicative part is not clearly described as indicated by the “ $\approx$ ” sign. Failure to consider this multiplicative part could lead to inaccurate estimation of the rainfall extremes (Nguyen and Pandey, 1994). Hence, in the present study, the basic multifractal equation can be written as a strict equality equation as follows:

$$\Pr(R_\lambda > \lambda^\gamma) = p(\gamma) \lambda^{-c(\gamma)} \quad (11.3)$$

in which $p(\gamma)$ represents the multiplicative part of the proposed model. It can be readily seen that, in order to use this model in practice, it is necessary to estimate the two parameter functions $c(\gamma)$ and $p(\gamma)$ using the available rainfall data in the study region.

11.3 Application

In the following, to assess the descriptive capability of the proposed scale-independent probability model (Equation 11.3) hourly rainfall records from a network of seven recording raingages (L'Assomption, Ste-Anne-de-Bellevue,

Oka-La-Trappe, Dorval, McGill, St-Jerome, and Ormstown) in the Montreal region (Quebec, Canada) are used. The record length varies from five years to 32 years, and only rainfalls during summer seasons (May to September) are considered to ensure the temporal homogeneity of the data. The codimension function $c(\gamma)$ as required by Equation 11.2 is estimated using the Double Trace Moments (DTM) technique (Lavallée, 1991). This estimation method, which is based on the scaling property of the statistical moments of the rainfall processes, is preferable to the Probability Distribution Multifractal Scaling (PDMS) method (Lavallée, 1991) because it allows for direct estimation of the two parameters α and C_1 , and the derived $\gamma - c(\gamma)$ curve is not limited by the data sample size considered.

Figure 11.1 shows the estimated $\gamma - c(\gamma)$ curves using daily rainfall data from five stations Dorval, McGill, L'Assomption, Ste-Anne-de-Bellevue, and Oka-La-Trappe. The closeness of these curves indicates that a single regional $\gamma - c(\gamma)$ relationship can be developed for the whole study area using all available daily rainfall data in the region. Figure 11.2 indicates also a very good closeness of the $\gamma - p(\gamma)$ relationships for various sites. Further, this graph suggests that the $\gamma - p(\gamma)$ relation can be approximated by a simple linear regression function: $p(\gamma) = b_0 + b_1 \gamma$. In summary, it can be seen that these gages exhibit a similar scaling behaviour. Accordingly, a single regional model (Equation 11.3) can be determined for all gages within the climatically homogeneous region considered. Note that the rainfall data at Ormstown station are not used in the development of the regional model in order to assess its predictive ability in the estimation of rainfall distribution for an ungaged site.

Figure 11.1 At-site and regional $\gamma - c(\gamma)$ relationships for the Montreal region.

Figure 11.2 At-site and regional γ - $p(\gamma)$ relationships for the Montreal region.

Figure 11.3 Quantile-quantile plot for 1-hour rainfall resolution at Dorval station.

For purposes of illustration, Figure 11.3 shows the quantile-quantile plots of the observed and estimated 1-hour resolution rainfalls at Dorval station based on the proposed at-site and regional models. As can be seen, the good agreement between observed and estimated rainfall amounts (as compared to the 45-degree straight line representing the perfect estimate of rainfall amounts) demonstrates the adequacy of the at-site model given that its parameters were estimated using rainfall data at daily time scale. Further, the closeness between at-site and regional rainfall estimates as shown in Figure 11.3 indicates the descriptive ability of the regional model proposed. Finally, Figure 11.4 shows the good predicting performance of the regional model in the estimation of 1-hour rainfall amounts at Ormstown station.

Figure 11.4 Quantile-quantile plot for 1-hour rainfall resolution at Ormstown station.

11.4 Conclusions

It has been shown that the probability distributions of rainfall over a wide range of time scales at a single site can be described by one resolution-independent probability model based on the theory of multifractal multiplicative cascade process. Results of the data analysis in the present study have provided some credence to this hypothesis. Further, the illustrative application of the

proposed model has indicated its descriptive and predictive capabilities. Finally, for the Montreal region, it was found that a single regional probability model can be used to describe the scaling nature of rainfall distributions within the whole area. The range of the temporal scales of aggregation is from one hour to the order of 16 days. Further studies are planned to apply the proposed methodology to data available in other geographic regions.

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