

Chapter 15

Development and Application of a Full Phosphorus Cycle Water Quality Model to Lake Champlain

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Lake Champlain is one of the largest bodies of freshwater in the United States, stretching almost 200 km long yet only 19 km wide. Its drainage basin of 10450 km² includes portions of Vermont, New York, and Quebec. As with many other water bodies, man's presence on and around the lake and in its drainage basin has had negative impacts. The most pressing water quality problem today is excess phosphorus loading which contributes to eutrophication and subsequent degradation of lake water quality.

The water quality model described here was developed to study the dynamics and kinetics of the distribution and concentration of nutrients in the lake. The development focused on the phosphorus cycle, including phytoplankton population simulation, and the estimation of lake response to phosphorus loadings. A comprehensive water, chloride, (as a conservative tracer) and phosphorus mass balance dataset developed through the joint effort of the Vermont Department of Environmental

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Conservation (VTDEC) and the New York State Department of Environmental Conservation (NYSDEC), for the two year period between March 1990 and February 1992, is used to provide flow and loading information for model input, and mean *in-situ* concentration information for comparison with model calculations.

The model conservation of mass equations are solved using a multiply-connected control volume approach in three dimensions and in time. This is a sophisticated version of the standard segment model approach. The model development also presents a unique approach to the determination of inter-segment mixing exchange, based entirely on a conservative tracer data set, chloride in this case. The method presented eliminates the trial-and-error and tuning necessary in other modeling approaches.

15.1 Introduction

To address the water quality issues of Lake Champlain, a federal act was signed into law in November 1990, adding the lake to the list of ten waterbodies eligible for lake water quality demonstration programs. The goals of this program are to preserve or enhance water quality by controlling point and non-point sources of pollution with the development of appropriate methods, technologies, and strategies. A specific goal is to develop a comprehensive plan for the lake to prevent and control pollution as well as restore and maintain the water quality, indigenous populations, and multiple uses of the lake.

An important step in the development of a comprehensive plan for lake management is to study and understand the underlying physical process that affects the levels and distribution of pollutants in the lake. The objective of the present study was to develop a model to simulate transport, mixing and hydrodynamic dispersion for estimation of spatial and temporal distribution and transport of water quality constituents and biological measures of importance. As part of the project, a hydrodynamic model was developed to study the circulation, mixing and transport phenomena in the lake, with a particular focus on the wind-driven response of the lake and the development of large-amplitude internal seiche reported in stratified conditions (Mendelsohn et al., 1994). The focus here will be on the water quality model development and application.

15.2 Phosphorus Model Development

The issue of whether a simplified water quality model can accurately portray the processes of a system is dependent upon the physics and chemistry of the water body. There has been, however, a long and successful history of box model applications to estuarine (Officer, 1980; Thomann and Mueller, 1987 and Swanson and Jayko, 1988) and lake problems (Chapra and Reckhow, 1983; DiToro and Connolly, 1980). The approach taken here is a sophisticated version of a simple concept and provides hydrodynamic information about advective and dispersive flows in three dimensions deduced from the distribution of a conservative tracer, chloride, given flow rates and loads to the system. Although the conservation of momentum equation is not involved directly in the approach taken, the effects of current movement and circulation are accounted for in an integrated sense.

The model is based on the box model methodology presented by Officer (1980) linked with the phosphorus kinetic rate equations taken from WASP5 (Ambrose et al., 1993). This methodology has a number of desirable capabilities: both conservative and nonconservative constituents can be examined; with two layers, both advective and nonadvective exchange estimates can be made; and the approach is sufficiently general to handle complex geometries.

The assumptions implicit in this type of model are the following:

1. The lake can be divided into a series of boxes which enclose volumes of constant properties such as chloride and phosphorus concentration.
2. The time-independent conservation equations for water and the tracer (chloride) can adequately describe the flow. No momentum equations are used.
3. Exchanges occur only at box faces, which allows a finite difference representation of the equations.
4. Constituent mass transport can be described by a conservation equation including loss relations, sources, sinks, and exchanges.

The box model requires that the lake be represented by a number of boxes of known area, volume, river flow and tracer concentration. Three equations are then solved for each box: conservation of water, chloride and constituent, to give the exchanges between boxes and the constituent concentration within the box.

One unique feature of this model is that the exchange rates are determined directly from observed chlorides. This approach seeks to eliminate the "trial and error" tuning usually used.

15.3 Exchange Coefficients

Following the approach of Officer (1980), longitudinal exchange coefficients (E) rather than dispersion coefficients are used to describe the dispersive exchange. Nonadvective chloride transfer is therefore represented by a term $E_{ij}S_i$, i.e.

$$E_{ij}S_i = \left(\frac{K_x A}{\Delta x} \right) S_i \quad (15.1)$$

where:

- E_{ij} = longitudinal exchange coefficient from box i to box j
- S_i = salinity of box i
- K_x = longitudinal dispersion coefficient
- A = cross-sectional area
- x = longitudinal distance

This relationship assumes that the exchange coefficient E_{ij} from box i to box j is equal to the exchange coefficient E_{ji} from box j to box i.

A conceptual diagram of the hydrodynamic exchanges occurring across the vertical and horizontal faces of a box is shown in Figure 15.1. Several naming conventions are followed. E and Q represent nonadvective and advective exchanges, respectively, while R is the net river flow. The ratio α is used to proportion the net river flow between the upper and lower layers: αR is the river flow in the upper layer, $(1-\alpha)R$ is the flow in the lower layer. The use of α is a generalization of the work done by Officer (1980) which assumes one-half the river flow in each layer. The chloride in each box is represented by S. A prime (') indicates that the parameter represents conditions in the lower layer of the box. Lettered subscripts indicate the face across which the exchange occurs; e.g. E_{ij} is the nonadvective exchange between box i and box j. A subscript v indicates a vertical exchange between the upper and lower layers. The box under consideration is box i; adjacent boxes are numbered a1 to aN depending on the number of connections, but may be thought of in a finite difference

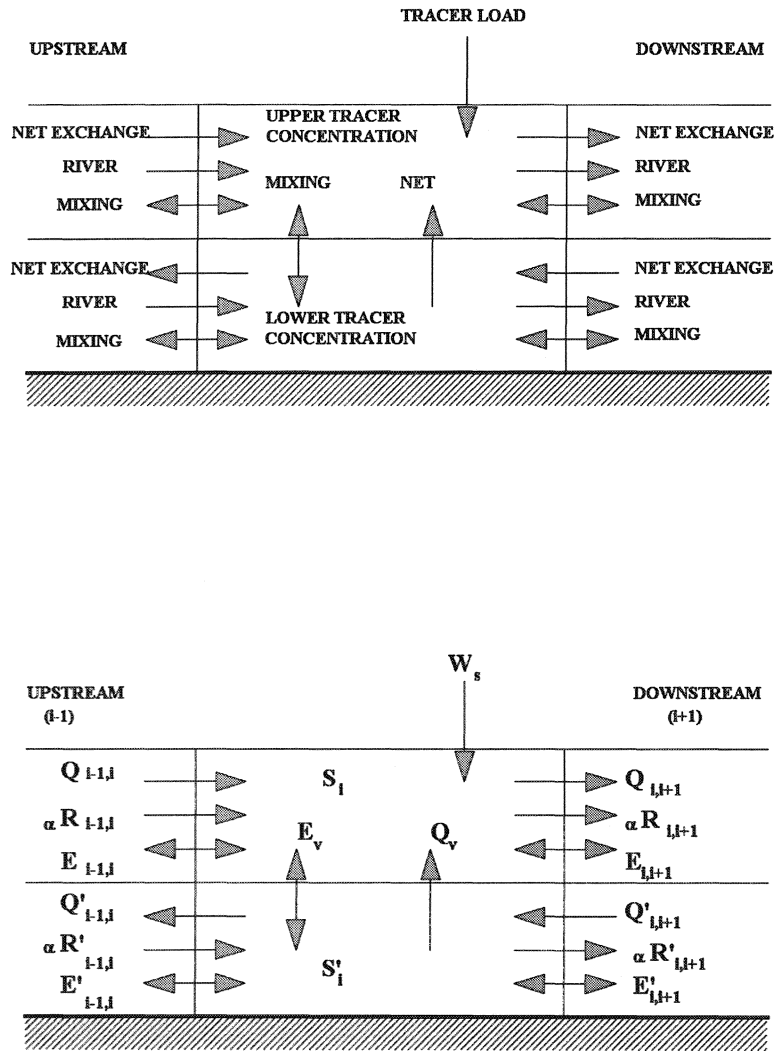


Figure 15.1
Conceptual diagram of hydrodynamic exchanges in the box model.

sense where upstream is $i-1$, downstream $i+1$ for convenience. There are no restrictions on the number of adjacent boxes a box may have. This is true since there is no directionality in the model, only flow in and out which is proportioned *a priori*. Flow into a box is considered positive; flow out of the box is negative. To simplify equation formulation a parameter δ is introduced to specify flow direction. If flow is into a box then $\delta=1$; for flow out $\delta=0$.

Invoking the tracer conservation equation for box i , upper layer, (using a 1-D example for clarity) yields

$$\begin{aligned} V_i \frac{dS_i}{dt} = & W_s + (\alpha R_{i-1,i} + Q_{i-1,i})S_{i-1} - \\ & (\alpha R_{i,i+1} + Q_{i,i+1})S_i + E_{i-1,i}(S_{i-1} - S_i) + \\ & E_{i,i+1}(S_{i+1} - S_i) + E_v(S'_i - S_i) + Q_v S'_i = 0 \end{aligned} \quad (15.1)$$

if the river flow is from box $i-1$ through box i to box $i+1$. The introduction of additional tracer material into a box is accounted for by the load W_s .

This equation is then rearranged, with the flow direction parameter δ incorporated, to solve for E and E' on each side of the box (remembering that $E_{i,i+1} = E_{i+1,i}$). In general for an N -sided box, the exchange between box i and j can be written:

$$\begin{aligned} E_{i,j} = & \frac{W_i + \sum_{a=1, a \neq i}^N \left\{ S_a \left[\delta_{i,a} (\alpha R_{i,a} + Q_{i,a}) + E_{i,a} \right] - S_i \left[\delta_{i,a}^m (\alpha R_{i,a} + Q_{i,a}) + E_{i,a} \right] \right\}}{(S_i - S_j)} + \\ & \frac{(\alpha R_{i,j} + Q_{i,j})(S_i \delta_{i,j} - S_i \delta_{i,j}^m)}{[S_i - S_j]} \end{aligned} \quad (15.2)$$

where:

$$\delta_i^m = (1 - \delta_i).$$

The equation for E and E' are now in terms of S and R which are known inputs. Solutions for Q , Q' , Q_v and E_v are then found which are also in terms of S and R . Starting at the upstream box, the exchange coefficients are found at each of the downstream faces of each successive

box through the system. This may need to be iterated several times to reach convergence for systems with more than one upstream direction.

15.4 Conservation of Constituent Mass

To solve for a constituent concentration C , in each box, the conservation of constituent mass equation is written for each of the constituents in both the upper and lower layers. The EPA WASP model kinetic rate equations are then integrated into the box model formulation to simulate the phosphorus cycle term interaction. Additional terms are also included to allow for flux input (loading) output and constituent settling as shown in Figure 15.2. For the upper layer of box i , the steady-state constituent conservation equation becomes

$$\begin{aligned}
 V_i \frac{dC_i}{dt} = & (\alpha R_{i,j-1} + Q_{i,j-1}) [\delta_{i-1} C_{i-1} - \delta_{i-1}^m C_i] + E_{i,j-1} (C_{i-1} - C_i) - \\
 & (\alpha R_{i,j+1} + Q_{i,j+1}) [\delta_{i+1} C_i - \delta_{i+1}^m C_{i+1}] + E_{i,j+1} (C_{i+1} - C_i) + \\
 & Q'_v C_i + E_v (C'_i - C_i) + W_i - BV_i - \lambda C_i V_i - A_w C_i = 0
 \end{aligned} \quad (15.3)$$

The additional terms are defined as:

- W_i = flux input or output of constituent (mass/time)
- BV_i = uptake of constituent, where
- B = mass uptake rate (mass/volume/time)
- V_i = volume of the upper layer of the box
- $\lambda C_i V_i$ = exponential growth/decay of constituent, where
- λ = exponential growth/decay rate (1/time)
- V_i = volume of the upper layer of the box
- $A_{iw} C_i$ = settling of constituent, where
- A_i = cross-sectional area between upper and lower layers of the box
- W = settling velocity (distance/time)

For simplicity the area over which settling occurs from the upper to the lower layer is assumed to be equal to the horizontal surface area of the box. The same is true for settling from the lower layer to the sediment layer.

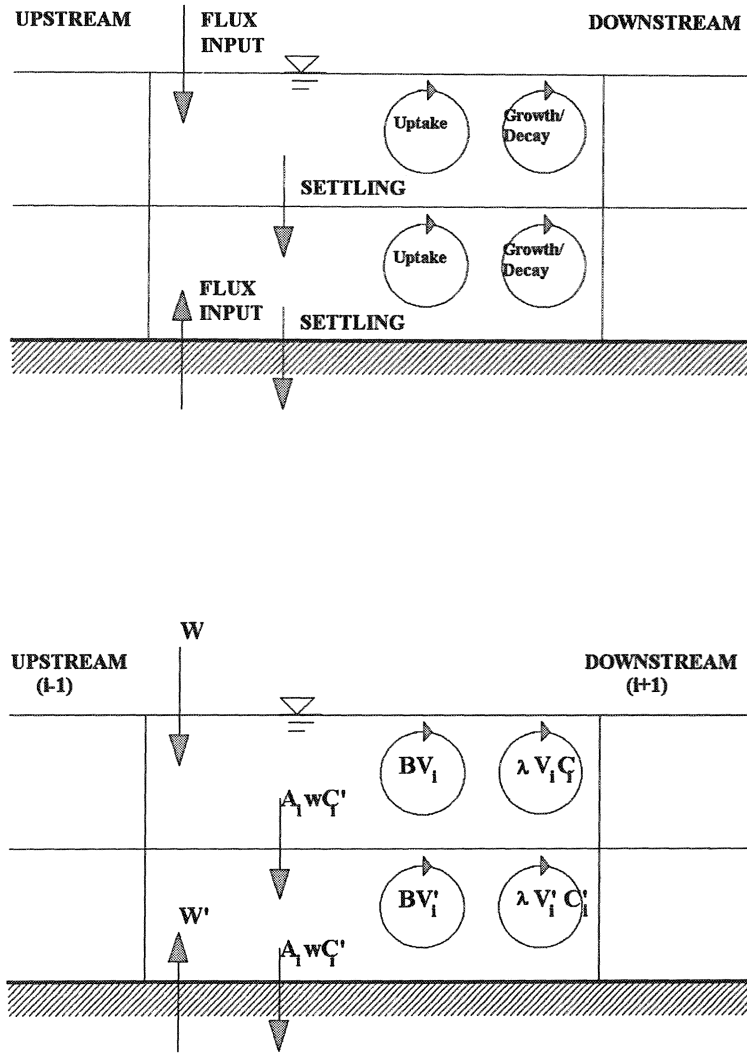


Figure 2
Conceptual diagram of non-exchange mechanisms affecting concentration.

Equation 15.3 can be recast into a form more amenable to solution by grouping the terms associated with the concentration in each adjacent box. Again for our one-dimensional example the conservation equation becomes

$$\begin{aligned}
 & (\delta_{i-1}\alpha R_{i,i-1} + \delta_{i-1}Q_{i,i-1} + E_{i,i-1})C_{i-1} \\
 & + [\delta_{i+1}^m\alpha R_{i,i+1} + \delta_{i+1}^mQ_{i,i+1} + E_{i,i+1}]C_{i+1} \\
 & + (Q_v + E_v)C_i' + W_i - BV_i \\
 & - [\delta_{i-1}^m\alpha R_{i,i-1} + \delta_{i-1}^m \cdot Q_{i,i-1} + E_{i,i-1} \\
 & + \delta_{i+1}\alpha R_{i,i+1} + \delta_{i+1}Q_{i,i+1}E_{i,i+1} \\
 & + E_v + \lambda V_i + A_{iw}]C_i = 0
 \end{aligned} \tag{15.4}$$

which is of the form:

$$AC_{i-1} + BC_{i+1} + CC_i' + D = EC_i \tag{15.5}$$

where A through E are algebraic combinations of river flow, advective exchanges, and the addition flux terms described above. It can be seen that the concentration in a box is affected by the concentration in each adjacent box, in addition to flux inputs and losses of the constituent. Equation 15.5 is used iteratively to solve for constituent concentrations using a successive-over-relaxation technique for steady state simulations and an explicit time-stepping method for transient cases (Roache, 1976).

The constituent kinetics are all contained in terms D and E of Equation 15.5. The model has been set up to solve for four constituents: a tracer, total organic phosphorus, total inorganic phosphorus and phytoplankton. For the tracer constituent the simple kinetics described above are used. For the phosphorus and phytoplankton the term D is replaced by the WASP5 kinetic rate equations.

15.5 Phosphorus Cycle Kinetics

Although suitable for a first-order estimation of lake eutrophication, with the use of total phosphorus the objection remains that a large portion

of the input total is associated with the particulate matter that rapidly settles out after entering the lake. It is clear that this fraction of the total would have little effect on mid-lake productivity. To analyze this and other effects that influence phosphorus concentrations and lake trophic status such as phytoplankton-phosphorus interaction, a model needs to differentiate between the various forms of phosphorus.

The phosphorus kinetics used for this model are taken from the Water Quality Analysis Simulation Program, WASP5 (Ambrose et al., 1993) which may be referred to for more detail. The phosphorus cycle models three variables: organic phosphorus, inorganic (orthophosphate) phosphorus and phytoplankton phosphorus. The organic and inorganic phosphorus variables are partitioned into dissolved and particulate fractions based on the total and a designated fraction for each. Both organic and inorganic phosphorus are coupled to phytoplankton through basic biological functions. Dissolved inorganic or “available” phosphorus is taken up by phytoplankton growth and returned from the phytoplankton biomass to both dissolved and particulate organic and dissolved inorganic phosphorus through respiration and mortality. A temperature-dependent fraction of the organic phosphorus also undergoes a mineralization or bacterial decomposition into dissolved inorganic phosphorus. The set of coupled phosphorus cycle rate equations are presented below with a brief description of the variables.

The phytoplankton phosphorus rate equation (i.e., the change in phosphorus as phytoplankton biomass with time) may be written as (Ambrose et al., 1993):

$$\frac{\delta(C_4 a_{pc})}{\delta t} = \underset{\text{growth}}{G_{p1} a_{pc} C_4} - \underset{\text{death}}{D_{p1} a_{pc} C_4} - \underset{\text{settling}}{\frac{v_{s4}}{D} a_{pc} C_4} \quad (15.6)$$

where:

- C_4 = phytoplankton population, (mg Carbon/l)
- a_{pc} = phosphorus to carbon ratio, (mg P/mg C)
- t = time, (days)
- G_{p1} = specific phytoplankton growth rate, (1/day)
- D_{p1} = phytoplankton loss rate, (1/day)
- v_{s4} = settling velocity, (m/day)
- D = segment depth, (m)

The growth rate is specified by a fixed maximum value which is a function of temperature, light limitation and phosphorus limitation. The

light limitation factor X_{RI} takes into account the seasonal, depth and turbidity light attenuation and photoinhibition effects on phytoplankton population growth. WASP5 offers a choice of two similar light modeling formulations. For this application the formulation developed by DiToro et al. (1971) was chosen; it averages conditions over a given depth and over a fixed interval of time (one day in the present units).

WASP models the phytoplankton death term as a function of respiration, death from parasitization and herbivorous zooplankton grazing. Lacking data on these processes, the temperature-variable respiration rates were selected from the manual (Ambrose et al., 1993) and all other death components were incorporated into a single term (i.e., grazing rate was set to zero).

The kinetic rate of change of organic phosphorus in the system is given by (Ambrose et al. 1993):

$$\frac{\delta C_8}{\delta t} = \underbrace{D_{P1} a_{PC} f_{OP} C_4}_{\text{phytoplankton death}} - \underbrace{K_{83} \theta_{83}^{T-20} \left(\frac{C_4}{K_{mPC} + C_4} \right)}_{\text{mineralization}} \cdot C_8 - \underbrace{\frac{v_{S8}}{D} (1 - f_{D8})}_{\text{settling}} C_8 \quad (15.7)$$

where:

- C_8 = organic phosphorus concentration, (mg/l)
- f_{OP} = fraction of dead and respired phytoplankton recycled
- K_{83} = dissolved organic phosphorus mineralization rate at 20°C, (1/day)
- θ_{83} = temperature coefficient
- K_{mPC} = half saturation constant for phytoplankton limitation of phosphorus recycle, (mg C/l)
- v_{S8} = organic matter settling velocity, (1/day)
- f_{D8} = fraction dissolved organic phosphorus in the water column

The remainder of the constants and variables are as previously designated.

Finally, the inorganic phosphorus kinetic rate of change equation is given by (Ambrose et al., 1993):

$$\frac{\delta C_3}{\delta t} = \underbrace{D_{P1} a_{PC} (1 - f_{OP})}_{\text{death}} C_4 + \underbrace{K_{83} \theta_{83}^{T-20} \left(\frac{C_4}{K_{mPC} + C_4} \right)}_{\text{mineralization}} - \underbrace{G_{P1} a_{PC} C_4}_{\text{growth}} - \underbrace{\frac{v_{83}}{D} (1 - f_{D3})}_{\text{settling}} C_3 \quad (15.8)$$

where:

- $(1-f_{op})$ = fraction of dead and respired phytoplankton recycled to the phosphate phosphorus pool
- v_{s3} = inorganic sediment settling velocity, (m/day)
- f_{D3} = fraction dissolved inorganic phosphorus in the water column

15.6 Application to Lake Champlain

The control volume approach assumes that the region under consideration, the entire Lake Champlain water body in this case, can be segmented into a number of boxes in which the physical characteristics remain approximately uniform throughout. For a two-layer system the thermocline is used to differentiate between the upper and lower layers. Since the calculated mixing exchange coefficients are assumed to be an integrated value over time, *mean* values of chloride concentration, load and freshwater flow must be specified for each box.

This study relied heavily on the comprehensive water, chloride and phosphorus mass balance data set collected and analyzed by the combined efforts of the NYSDEC and the VTDEC and presented in Lake Champlain Diagnostic-Feasibility Study Interim (1992) and Final Reports (1993). The reader is referred to the study report for a complete description of the data set. The VTDEC/NYDEC data set has been analyzed and averaged over two different periods. Initially the data was analyzed over the entire two-year survey, but, based on comparison with long-term stream-flow gage data, it was determined that the calendar year 1991 better represented the long-term annual mean flows. The 1991 calendar year was therefore selected as a hydrologic “base year” for determining long-term average conditions and for subsequent use in their phosphorus modeling effort.

In order to compare results with and build on previous studies, the lake segmentation developed by the VTDEC was used. This segmentation divides the lake into thirteen sections based on phosphorus criteria (VT, NY and Que, 1993 Lake Champlain Water Quality Agreement) and present conditions. The geographic location and area/coverage of each segment is shown in Figure 15.3. A schematic of the lake segmentation indicating connections between segments, river inputs and net flow directions is shown in Figure 15.4. Table 15.1 presents the model segment morphometric data developed by the VTDEC.

Water, chloride and phosphorus sources to Lake Champlain consist of rivers, municipal and industrial waste water treatment facilities (WWTF), ungaged drainage areas, groundwater inflow and direct precipitation. Outflow from the lake consists of the Richelieu River outlet, direct water withdrawals, evaporation and groundwater outflow. The water, chloride and phosphorus mass balance results from the NYDEC/VTDEC two-year survey dataset are presented in Table 15.2.

Mean flow rates and chloride and phosphorus concentration data were determined and tabulated for the entire two-year survey and for the hydrologic “base year”. These flow rates and concentrations were then grouped geographically according to which segment would receive their input. The flow rates for a particular segment were summed to create a single input flow rate per segment. The loads were determined by first multiplying the particular source mean flow rate by the mean concentration to create a mean load by source. The individual loads were then summed according to their segment grouping to create a single mean load for each segment. The results for both the two-year survey and the base year are presented in Tables 15.3 and 15.4, respectively. The loads generated for chloride and total phosphorus (TP) are straight forward, as described above. Loads for total organic phosphorus (TOP) and total inorganic phosphorus (TIP) were calculated from the TP values in the lake-wide data set and a partitioning value calculated from the more detailed phosphorus analyses at three vertical profile stations.

The 52 station *in-situ* data set was also processed by the VTDEC to produce time-weighted mean concentrations of chloride and total phosphorus by segment for the two-year survey. The method used compensated for the greater number of samples taken in the summer months so that these values would not skew the data. The results are repeated here in Table 15.5. The chloride concentrations are used to calibrate the exchange coefficient calculations, as are the total phosphorus used to calibrate the phosphorus cycle kinetics.

15.7 Exchange Rate Coefficients

The mean flows and chloride loads were used, along with the mean *in-situ* chloride concentrations, to calculate the mixing exchange coefficients at the segment interfaces. A comparison of the results for the two year survey load means to the VTDEC exchange calculations is shown in Figure 15.5(a) and to the values for the hydrologic base year in Figure

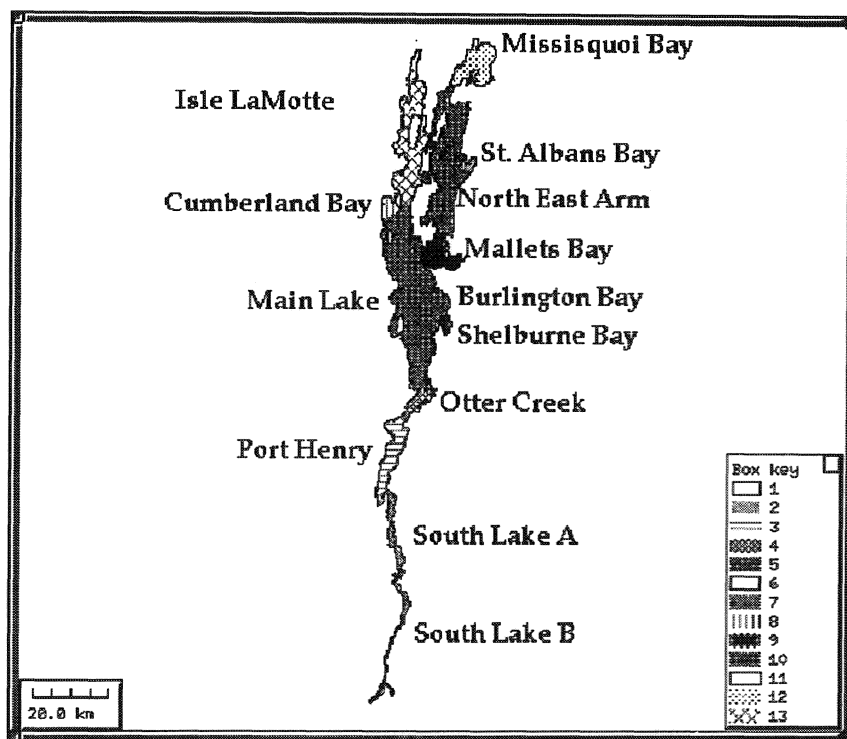


Figure 15.3
Phosphorus model segmentation geography.

15.5(b). The segment designation indicates that the exchange value is given for the downstream (river flow out) face of that segment. Segment 9, Malletts Bay has two outlets, one to the west adjoining the main basin and one to the north adjoining the northeast arm (see Figure 15.3 and 15.4). The river flow was partitioned between the two outlets in Malletts Bay following the results of data given in Myer and Gruending (1979) and the VTDEC analysis which reports that approximately 16% of the flow goes north to the northeast arm and the remainder flows to the main lake.

It is clear that the values calculated for the exchange coefficients by the ASA model are very similar to those of the VTDEC model although two quite different methods for the exchange calculation are used. The ASA model relies entirely on the flow, load and concentration data whereas the VTDEC model necessitates the determination of a diffusion parameter, appropriate mixing lengths and appropriate cross-sectional areas for exchange as well.

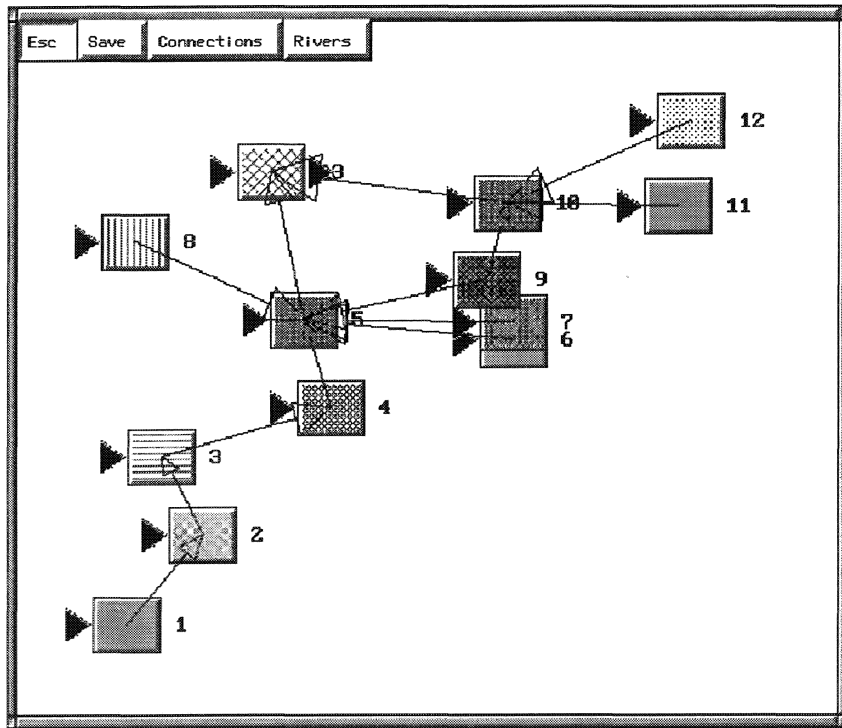


Figure 15.4
Schematic of the model segmentation, connections and net flow path.

15.8 Phosphorus Cycle Kinetics Calibration

The constituent concentration model as adapted for modeling the phosphorus cycle computes the response of three dependent variables, organic phosphorus, inorganic phosphorus and phytoplankton to a given loading.

The hydrologic base-year inflow data-set was used to calibrate the various coefficients and parameters described above for phosphorus cycle kinetics, as it better represents the long-term mean inflow and loading conditions found in the lake. It should be kept in mind that the model as developed is capable of simulating more variables than we have data to support. A number of processes have therefore been lumped together under common values which, under real conditions, would be more fully

Table 15.1
Lake model segment morphometric data, and listing of lake sampling stations
within each segment (VTDEC/NYDEC, 1992).

Segment	Area (km ²)	Volume (km ³)	Mean Depth (m)	Length (km)	Lake Sampling Stations
1. South Lake B	5.79	.0078	1.35	20.1	1, 2
2. South Lake A	43.27	.125	2.89	33.5	3, 5
3. Port Henry	75.55	1.463	19.36	20.1	6, 7
4. Otter Creek	28.49	.955	33.52	10.1	8, 9
5. Main Lake	414.14	16.787	40.53	47.0	10-15, 17-20, 22, 23, 26, 27, 28, 31
6. Shelburne Bay	9.62	.140	14.55	3.4	16
7. Burlington Bay	5.51	.063	11.43	2.0	21
8. Cumberland Bay	10.75	.063	5.86	3.4	33
9. Malletts Bay	55.06	.722	13.11	6.7	24, 25
10. Northeast Arm	248.25	3.380	13.62	33.5	29, 30, 34, 35, 37, 38, 39, 43, 45
11. St. Albans Bay	7.21	.023	3.19	3.4	40, 41
12. Missiquoi Bay	89.94	.205	2.28	16.8	47, 48, 50, 51, 52
13. Isle LaMotte	185.59	1.892	10.19	40.3	32, 36, 42, 44, 46, 49
Total	1179.17	25.826	21.90		

expressed. An example of this is the herbivorous zooplankton grazing rate, for which no data was available.

After a series of preliminary runs were made using the base year data, a sense of the model's sensitivity for certain parameters was determined. At the same time a set of "base-case" parameters was arrived at, which represented no calibration. The base-case parameters are listed in Table 15.6. A list of the parameters and the range over which they were varied is presented in Table 15.7. The parameters chosen for variation

were selected with the intention of attempting to explain, or at least investigate, some of the anomalous behavior observed in the preliminary runs.

One measure of the success of a particular parameter selection is the comparison of computed to observed mean total phosphorus concentrations by segment. With the present model's ability to simulate the mean phytoplankton population, another measure for comparison is the chlorophyll *a* concentration means. For the comparison, a set of means was determined for the hydrologic base year as shown in Table 15.8 by segment. The data-set in Table 15.8 will necessarily have a bias towards the warm weather concentrations because no time-weighting was used to average the values and the data-set was taken between the months of April and November.

Variables considered include: total daily insolation, saturation light intensity, carbon-to-chlorophyll ratio in the phytoplankton, death rate and respiration rate of phytoplankton, fraction of phosphorus recycled

Table 15.2
Water, chloride, and total phosphorus budgets for Lake Champlain,
March 1990 - February 1992. (VTDEC/NYDEC, 1992).

Source	Water (hm/yr)	Chloride (mt/yr)	Phosphorus (mt/yr)
Inputs			
Gaged Tributaries	11,387	106,980	780
Ungaged Tributaries	552	4,838	29
Direct Wastewater Discharges	52	12,423	57
Direct Precipitation	1085	312	18
Groundwater Inflow	ND	ND	ND
Total Inputs	13,076	124,553	884
Outputs			
Outlet Flow	12,809	131,933	181
Water Withdrawals	46	549	1
Evaporation	735	0	0
Groundwater Outflow	ND	ND	ND
Total Outputs	13,590	132,482	182
Change in Storage	-701	-7,220	-10
Error/Retention	187	-709	712
(% of Total Inputs)	(1%)	(1%)	(81%)

ND = no data

Table 15.3
Two year survey, March 1990 - February 1992.

Segment	Box #	Flow (m ³ /s)	Chloride Load (g/s)	TIP Load (g/s)	TOP Load (g/s)	TP Load (g/s)
South Lake B	1	34.56	360.93	0.629	2.034	2.663
South Lake A	2	19.51	465.07	0.323	0.319	0.642
Port Henry	3	4.02	40.01	0.076	0.165	0.241
Otter Creek	4	51.94	499.80	2.281	2.894	5.175
Main Lake	5	107.84	1098.30	0.952	5.831	6.783
Shelburne Bay	6	2.40	69.88	0.356	0.278	0.634
Burlington Bay	7	0.21	18.92	0.159	0.213	0.372
Cumberland Bay	8	30.44	188.31	0.651	0.738	1.389
Malletts Bay	9	47.95	446.56	0.403	1.242	1.645
Northeast Arm	10	1.68	29.45	0.042	0.098	0.141
St. Albans Bay	11	1.93	73.52	0.118	0.212	0.330
Missisquoi Bay	12	63.74	487.76	1.487	5.100	6.587
Ilse LaMotte	13	14.46	154.73	0.359	0.594	0.954
Total		380.68	3933.23	7.837	19.718	27.556

the organic pool, phytoplankton settling rate, phosphorus settling rate (organic and inorganic together), riverine chlorophyll load and water exchange rates between segments within the lake.

The value for total daily solar radiation was determined for Burlington, Vermont by averaging the monthly average daily radiation over a one year period, (from NOAA meteorological station data; Duffie and Beckman, 1980). Water column light extinction was calculated from secchi depth data, again averaged for the base year, by station then by segment (Table 15.8). These values were corrected for phytoplankton self-shading based on the average chlorophyll concentrations and the self-shading formulation used by the WASP model.

Total phosphorus (TP) and chlorophyll *a* are the only variables consistently reported in the lake data. Thus these are the only variables available to which the model can be calibrated. The extent to which this

Table 15.4
Hydrologic base year, January 1991 -December1991.

Segment	Box #	Flow (m ³ /s)	Chloride Load (g/s)	TIP Load (g/s)	TOP Load (g/s)	TP Load (g/s)
South Lake B	1	26.31	291.87	0.421	1.361	1.782
South Lake A	2	14.41	428.75	0.294	0.242	0.536
Port Henry	3	2.81	31.18	0.068	0.090	0.158
Otter Creek	4	40.45	404.61	1.776	2.087	3.863
Main Lake	5	84.87	920.59	0.779	3.239	4.019
Shelburne Bay	6	1.86	61.47	0.300	0.203	0.503
Burlington Bay	7	0.20	18.66	0.273	0.098	0.372
Cumberland Bay	8	26.59	169.86	0.631	0.658	1.290
Malletts Bay	9	37.10	347.07	0.285	0.761	1.046
Northeast Arm	10	1.30	22.78	0.029	0.071	0.100
St. Albans Bay	11	1.43	58.16	0.124	0.139	0.262
Missisquoi Bay	12	54.19	415.07	1.274	3.997	5.271
Ilse LaMotte	13	12.46	132.89	0.328	0.536	0.865
Total		304.00	3302.97	6.582	13.483	20.066

limits the reliability of the calibration for any particular model parameter should be noted.

Based on the results of these sensitivity runs, a set of parameter value adjustments was selected to attempt calibration of the model for TP and chlorophyll *a*. Phytoplankton settling was reset to 0.15 m/day while organic and inorganic phosphorus settling was reset to 0.4 m/day. Though river chlorophyll *a* loads overall appeared to contribute poorly to model calibration, it was apparent that the only way to get the measured value of chlorophyll *a* found in the rapidly-flushed segment 1 was to add it in the rivers. A best-fit value was iteratively determined to be 12 µg/L chlorophyll *a* in the river to segment 1 only. Based on observations in VTDEC and NYSDEC (1993), the very low computed TP concentration in segment 11 was attributed to a non-equilibrium state in which sediments provide a net TP source in that segment. A value of 6.4 mg/m²/day

Table 15.5
Mean chloride and total phosphorus concentrations in each lake segment,
1990-1991. (VTDEC/NYDEC, 1992).

Segment	Chloride (mg/l)		Total Phosphorus (µg/l)	
	Mean	C.V.	Mean	C.V.
South Lake B	11.62	.042	57.55	.067
South Lake A	13.47	.045	33.88	.062
Port Henry	11.18	.007	14.97	.048
Otter Creek	10.72	.005	14.58	.050
Main Lake	10.61	.004	11.79	.025
Shelburne Bay	10.89	.008	15.09	.058
Burlington Bay	10.78	.006	13.34	.068
Cumberland Bay	10.18	.012	13.57	.067
Malletts Bay	9.43	.012	9.35	.059
Northeast Arm	9.29	.004	14.23	.024
St. Albans Bay	10.20	.010	23.71	.052
Missisquoit Bay	7.78	.019	35.24	.056
Isle LaMotte	10.33	.006	12.10	.027

C.V. = coefficient of variation of the mean

provided a best fit of TP and chlorophyll *a* together. The results of the calibration process are shown in Figures 15.6a and b. TP values are still computed to be somewhat low in segments 1, 2, 10, and 11 and very close or slightly high in the others. Chlorophyll *a* values are low in segments 3, 4, 5 and 10 while being very close or high in the others.

Several approaches to statistical evaluation of model fit to data are possible (McCutcheon et al., 1990). Two have been chosen here. The root mean square calculation (RMS) is evaluated as:

$$\left[\frac{\sum (C_D - C_M)^2}{n} \right]^{0.5} \quad (15.9)$$

where:

- C_D = the measured value (data)
 C_M = the model computed values
 n = the number of measurements taken and compared

Table 15.6
Phosphorus cycle kinetic rate equations terms for the "Base" case.

Phytoplankton Net Growth Terms	Exogenous Variables		Units
	Notation	Values	
Extinction Coefficient	K_e	Table 3.13	m^{-1}
Segment Depth	D	Table 3.2	m
Water Temperature	T	14.5	$^{\circ}C$
Fraction of day that is daylight	f	0.5	---
Total Daily Surface Solar Radiation	I_o	422	langleys/day
Zooplankton Population	Z	0	mgC/L
	Rate Constants		
Maximum Growth Rate	K_{1c}	2.0	day^{-1}
Temperature Coefficient	Q_{1c}	1.068	---
Phytoplankton Self-Light Attenuation	K_c	0.017	$m^2/mg\ Chl\ a$
Carbon-Chlorophyll Ratio	Q_c	35	---
Saturating Light Intensity	I_s	300	langleys/day
Half Saturation Constant for Phosphorus	K_{mPg}	0.001	mgP/l
Endogenous Respiration	K_{1R}	0.125	day^{-1}
Temperature Coefficient	Q_{1R}	1.045	---
Settling Velocity	V_{S4}	0.25	m/day
Death Rate	K_{1D}	0.02	day^{-1}
Grazing Rate	K_{1G}	0	L/mgC-day

Table 15.8
Hydrologic base year data averaged by segment.

Segment	Chlorophyll <i>a</i> (mg/l)	Secchi Depth (m)	Extinction Coefficient	KESHD	Corrected Extinction Coefficient
1	9.97	0.39	4.33	0.34	3.99
2	6.65	1.12	1.52	0.25	1.27
3	4.14	3.63	0.47	0.18	0.29
4	3.00	4.50	0.38	0.14	0.24
5	2.21	5.38	0.32	0.11	0.20
6	3.33	4.74	0.36	0.15	0.21
7	3.06	5.16	0.33	0.14	0.19
8	2.06	4.43	0.38	0.11	0.28
9	2.14	5.31	0.32	0.11	0.21
10	3.67	4.85	0.35	0.16	0.19
11	7.12	2.54	0.67	0.26	0.41
12	9.53	1.64	1.04	0.33	0.71
13	2.59	4.80	0.35	0.12	0.23

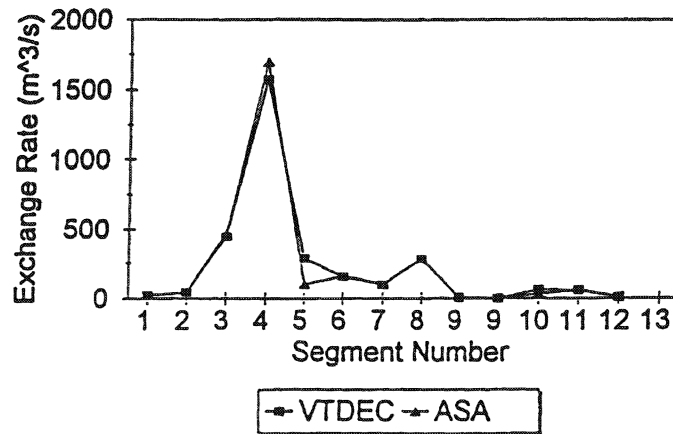
methods differ considerably. The method for determination of mixing exchange coefficients presented here make calculation relatively easy.

Total phosphorus concentration computations were made based on a “hydrologic base year” input dataset and in-lake concentrations. A sensitivity study on ten parameters was performed. Based on these sensitivity runs, a best-fit parameter set was iteratively chosen. Overall phosphorus computations compared well with observations.

The data for calibration was sparse in some areas, leaving room for additional calibration work in continuing studies. Additional information is needed on orthophosphate concentrations throughout the lake as well as in the tributary flows. Similarly, chlorophyll *a* concentrations and phytoplankton species and productivity data would be of interest. Finally, almost no data exists pertaining to the benthic processes.

Recommendations for future studies include comparisons and calibration of inter-segment mixing exchange and transport with hydrodynamic model calculations. Additionally, transient phosphorus model simulations could be performed to investigate the seasonal variations and other cyclical responses in phosphorus concentration in the lake.

Exchange Rate Comparison 2 Year Survey Data



Exchange Rate Calibration Hydrologic Base Year, Jan - Dec 1991

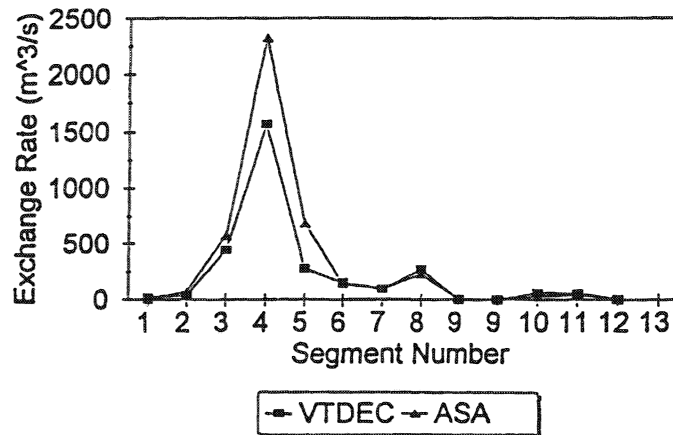


Figure 15.6a
Comparison of mixing exchange coefficient calculations:
(a) VTDEC calculations versus ASA calculations,
(b) Hydrologic base year versus 2 year data.

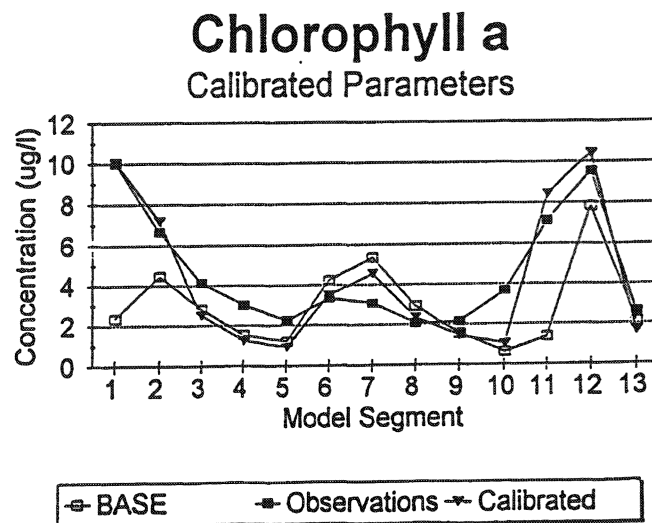
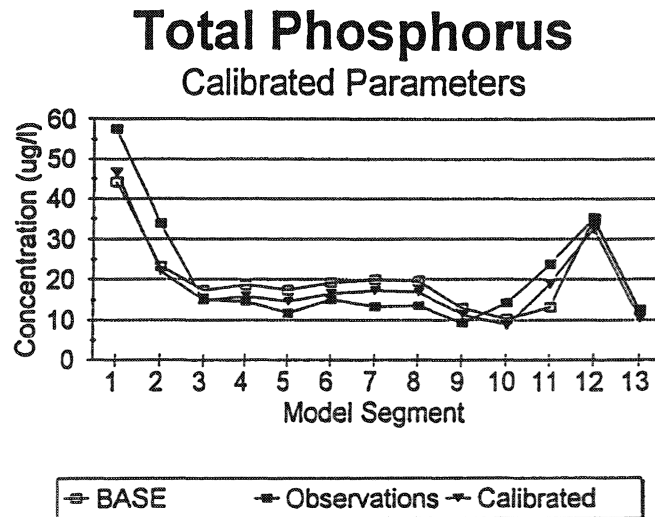


Figure 15.6b

(a) Total phosphorus concentrations computed by the model using the calibrated model parameters. Mean annual observed concentrations and base case computations are shown for comparison, (b) Chlorophyll *a* concentrations computed by the model using the calibrated model parameters. Mean annual observed concentrations and base case computations are shown for comparison.

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