Chapter 10

Water Quality Control under Uncertainty

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Water quality of the effluent from a treatment plant has random variations. In addition, the computed quality of receiving waters is affected by random noises due to uncertainty in initial conditions, non-point source loading, parameters that define decay, reaeration, and source terms such as photosynthesis activities, among others. The measurement noise is another random factor. It is safe to assume the underlying random characteristics of all but the effluents to be uncontrollable when considering the design of a treatment plant. The proposed work presents a new design procedure that relates the means and coefficients of variation of effluent to the probability of meeting the water quality standards of the receiving waters. The proposed procedure is an alternative method to the Monte-Carlo method which normally requires thousands of simulations, thus making the method unattractive to large problems. The new method is more accurate than the first-order analysis which cannot consider the dynamic randomness in parameters. Numerical examples are presented to illustrate the proposed methodology.

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10.1 Introduction

Current practices in treatment plant design include concepts based on reliability (Tchobanoglous and Burton, 1991). Because the effluent quality is not deterministic (Bohnke, et al., 1983), it is necessary to consider design under uncertainty. However, design methods so far generally do not consider directly the uncertainty of water quality in the receiving bodies of water. Considering the receiving water body interactions when designing a treatment plant may produce an optimal design in terms of reduced costs (Rossman, 1989) subject to satisfying the receiving body water quality standards at a specified reliability. In this chapter a new design procedure is proposed that relates the mean and coefficient of variation of the effluent water quality to the minimum DO level in the reach that is achieved with, for example, 95% reliability.

Although it is possible to use Monte-Carlo methods, the number of simulations required for higher confidence levels is very large (Papoulis, 1991). When the number of water quality indicators modeled become many, the Monte-Carlo method becomes impractical. The alternative method of first-order analysis that computes the mean and variance of water quality indicators directly is not suitable when model parameters such as the first-order decay coefficient is considered a random process, that is, when model parameters randomly vary along the reach of the river (McBean et al., 1994). The proposed method is more suitable for treatment plant design problems because it combines the advantages of the first-order analysis method and the Monte-Carlo method.

10.2 Problem Description

Consider a single river reach with a treatment plant discharging at the head of the reach. For simplicity only the Carbonaceous Bio-Chemical Oxygen Demand (BOD) and Dissolved Oxygen (DO) are modeled. Consider the following set of differential equations (Zielinski, 1988) for BOD (variable x_1) and DO (variable x_2).

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + \mathbf{b} \quad (10.1)$$

where matrix

$$A = \begin{bmatrix} -k_1 & 0 \\ -k_1 & -k_2 \end{bmatrix}$$

where:

$$\begin{aligned} k_1 &= \text{the BOD (mg/l) decay rate (1/day)} \\ k_2 &= \text{the reaeration rate (1/day)} \end{aligned}$$

and

$$\mathbf{b} = \begin{bmatrix} s_1 \\ s_2 + k_2 D_s \end{bmatrix}$$

where:

$$\begin{aligned} s_1 &= \text{the net non-point source load of BOD in mg/l/day} \\ s_2 &= \text{the net non-point source of DO (mg/l/day)} \\ &\quad \text{including that part due to the photo-synthesis} \\ D_s &= \text{the saturation concentration of oxygen (mg/l)} \end{aligned}$$

The initial BOD and DO conditions at the head of the reach, the parameters of the model, and the source terms s_1 and s_2 all may be considered random. In particular, the parameters of the model k_1 and k_2 are assumed to vary randomly along the reach of the river. Because the effluent discharge is assumed to be at the head of the reach of the river, the mean and variance of the effluent discharge can be added to the mean and variance of the initial conditions. Equation (10.1) is a set of stochastic differential equations with non-smooth solutions. Therefore, stochastic calculus is required for the integration (Jazwinski, 1970).

For the purpose of this chapter, the means of k_1 and k_2 are assumed to be 0.5 and 0.65, respectively, with the step length in the stochastic numerical integration = 0.01. The values of s_1 , s_2 , and D_s are all assumed to be deterministic with values of 3.0, 0.5, and 10.0, respectively. Figure 10.1 presents sample DO profiles determined using the stochastic calculus for the initial condition of 10.0 and 8.5, respectively, for the BOD and DO values. The effective standard deviation of both parameters k_1 and k_2 is equal to 10% of their respective mean. By simulating such profiles many thousands of times for each possible initial condition, the minimum DO

Figure 10.1
Simulations of random DO profiles.

level in the reach that is exceeded 95% of the time can be determined. However, the proposed methodology directly estimates the required information at a considerably reduced computing time using the moment equations described below.

10.3 Proposed Methodology

Using Ito's stochastic calculus [Jazwinski (1970)], a set of moment equations that describe the mean and second moment of the solution of the stochastic differential equations in (10.1) can be derived. The following is the result under the assumptions of random initial conditions and independently distributed random first-order rate parameters; for further details with correlations and other randomness in source terms see Finney et al. (1983):

$$\frac{dy}{dt} = A_y y + b_y \quad (10.2)$$

where:

$$\begin{aligned} y^T &= [E(x_1), E(x_2), E(x_1^2), E(x_1 x_2), E(x_2^2)] \\ T &= \text{transpose} \\ E(.) &= \text{the expectation operator.} \end{aligned}$$

The matrix A_y is a 5 by 5 matrix whose rows (from row 1 to row 5) are given respectively as:

$$\begin{aligned} &[-k_1, 0, 0, 0, 0], \\ &[-k_1, -k_2, 0, 0, 0], \\ &[2s_1, 0, -2k_1 + \sigma_{k_1}^2, 0, 0], \\ &[s_2 + D_s k_2, s_1, -k_1 + \sigma_{k_1}^2, -k_1 - k_2, 0], \text{ and} \\ &\left[0, 2(s_2 + D_s k_2 - D_s \sigma_{k_1}^2), \sigma_{k_1}^2, -2k_1, 2k_2 + \sigma_{k_1}^2 \right] \\ &\text{and} \\ &b_y^T = [s_1, s_2 + k_2 D_s, 0, 0, D_s^2 \sigma_{k_2}^2] \end{aligned}$$

Therefore, the original set of stochastic differential equations has been reduced to a set of equivalent deterministic equations for the first and second moments of BOD and DO concentration profiles. Equation (10.2) can now be integrated numerically using any standard scheme to find the mean and variance of the solution profiles of the original problem. For problems with three or more differential equations, an automatic formulation methodology has been developed by Curi and Unny (1991).

10.4 Results and Conclusions

Figure 10.2 presents results from the moment equations for the same data above and for different BOD source means and coefficient of variations. On the y-axis the 95% reliable minimum DO achieved in the

Figure 10.2
The 95% reliable minimum DO versus BOD source variability.

reach is plotted against coefficient of variation in the source BOD in the x-axis. The different curves correspond to different mean values of source BOD, that is, of the effluent. It is clear that, for a given mean value of source BOD, increasing the coefficient of variation brings down the minimum DO. Also, for a given coefficient of variation, increasing the mean brings down the minimum DO. Both results are as expected. Suppose the 95% reliable minimum DO is 5 mg/l, and the expected coefficient of variation (which may be dictated by the available technology and cost) is 20%, then, in order to meet the water quality criteria in this particular example, the mean of the effluent BOD can not exceed about 9 mg/l. Further, the methodology can easily be extended to multiple reaches and multiple water quality criteria. It is also possible to derive functional relationships and use them in an optimization model which may be used to design treatment facilities for an entire region. The probability distribution of DO in the reach is assumed to be normally distributed. However, if there is a strong indication of non-normality, then the Thebycheff

inequality (Papoulis, 1991) may be used to estimate the reliability using the means and standard deviations of the DO values in the reach.

In conclusion, the proposed methodology enhances current treatment plant design techniques by taking into consideration the variability or uncertainty in both the effluent and the receiving water bodies. The water quality reliability criteria is also explicitly considered.

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